Potential-well bunch lengthening in electron storage rings

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Acknowledgements

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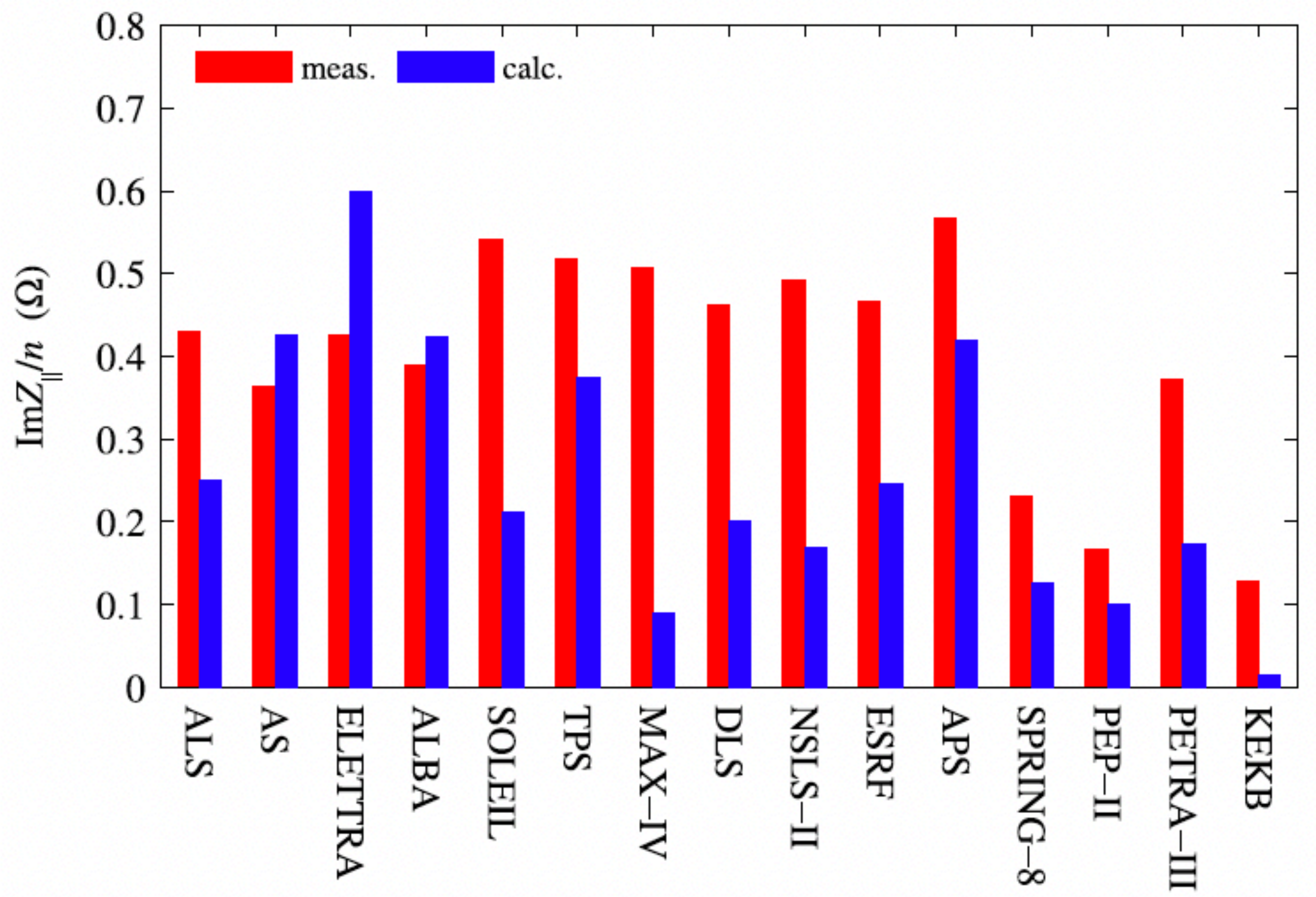
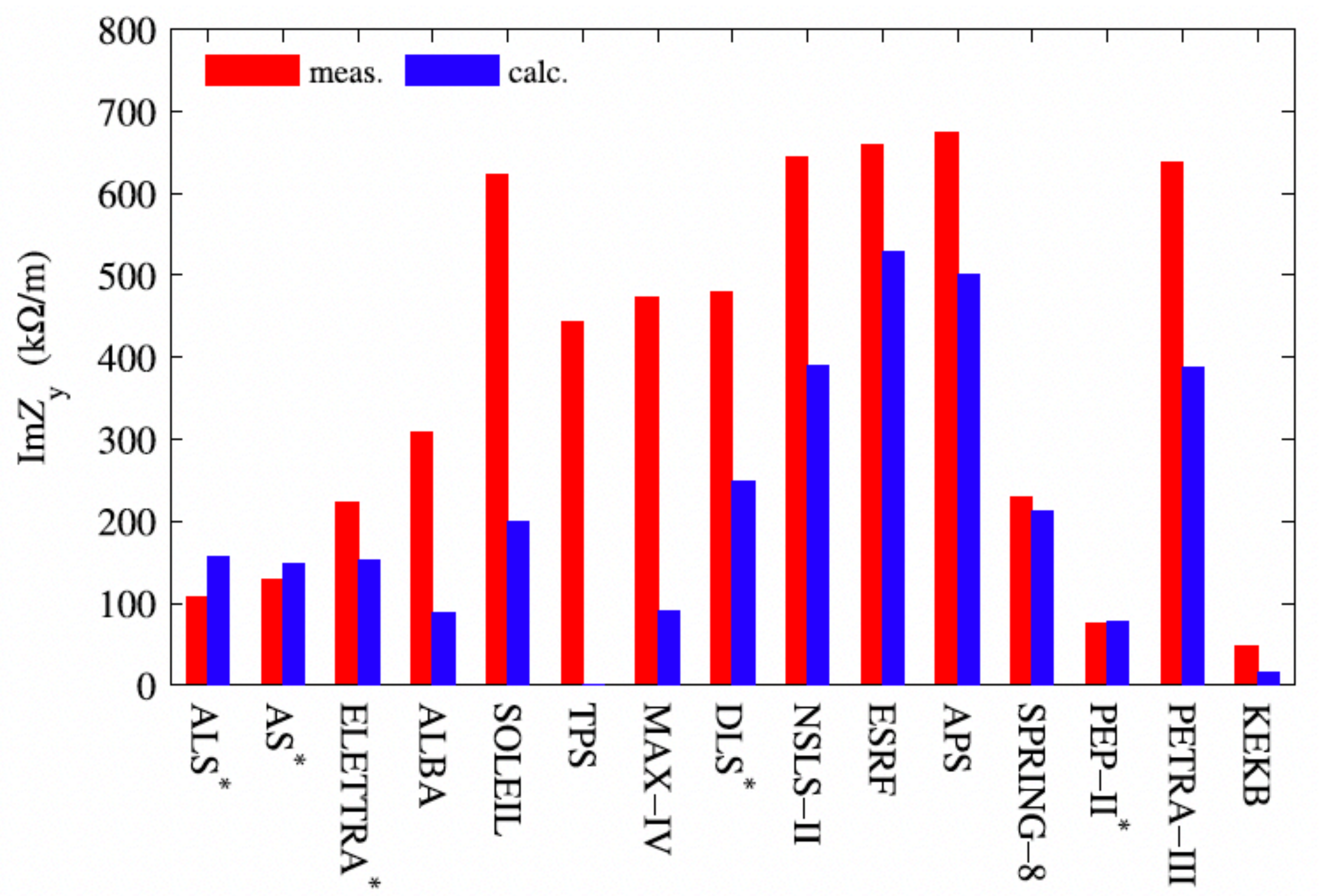
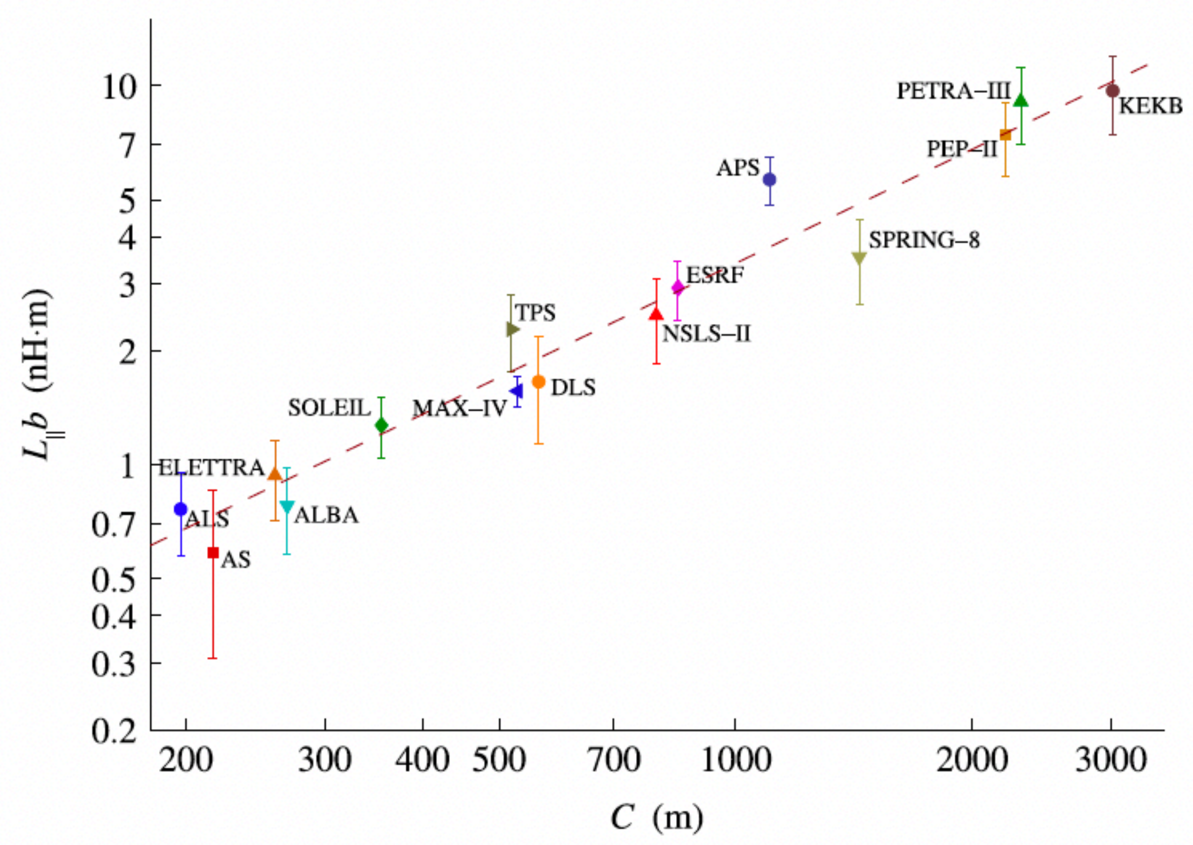
Motivation

- Discrepancy between impedance calculations and beam-based measurements
  - For several decades, theories and numerical tools for impedance calculations have been “well established”.
  - Techniques for experimental observations of impedance effects have also matured in parallel.
  - However, discrepancies remain in each accelerator project, to varying degrees [1].







Accelerator parameters.

	C (m)	E (GeV)	σ_{t0} [a] (ps)	σ_δ 10^{-3}	α 10^{-3}	ν_x	ν_y	β_x^{aver} (m)	β_y^{aver} (m)	$2a$ (mm)	$2b$ (mm)
ALS [18]	196.8	1.52	14	0.71	1.59	14.25	9.2	5.0	5.4	96	34
AS [22]	216	3	29	1.02	2.11	13.3	5.2	8.0	14.1	70	28
ELETTRA [19]	259.2	0.9	8	0.36	1.55	14.3	8.2	7.8	6.4	80	32
ALBA [23]	268.8	3	21	1.05	0.89	18.18	8.37	6.6	9.2	72	28
SOLEIL [24]	354.4	2.75	21	1.02	0.44	18.18	10.23	9.0	8.4	80	25
TPS [25]	518.4	3	10	0.89	0.24	26.18	13.28	8.9	9.0	70	32
MAX-IV [26]	528	3	49	0.782	0.306	42.2	14.28	3.8	7.0	22	22
DLS [27]	561.6	3	13	0.96	0.166	27.2	13.37	9.6	12.5	80	24
NSLS-II [28]	791.9	3	11	0.514	0.363	33.22	16.26	12.5	13.7	64	24
ESRF [20]	844.4	6	20	1	0.186	36.44	14.39	19.0	22.7	76	28
APS [20]	1104	7	24	0.96	0.228	36.2	19.27	13.5	16.0	84	34
SPRING-8 [20]	1436	8	12	1.09	0.146	40.14	18.35	17.0	18.1	70	40
PEP-II [29]	2200	3.1	34	0.77	1.23	24.51	23.61	15.9	12.1	110	76
PETRA-III [13]	2304	6	43	1.27	1.2	37.26	33.2	15.7	20.8	80	40
KEKB [29]	3016	3.5	13	0.727	0.32	45.51	43.58	13.1	14.2	94	94

Zotter’s equation [2]:

$$x^3 - x - \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta 0}^2(E/e)}\mathbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1} = 0$$

We must be cautious:
The model can be a source of discrepancies if its assumptions are violated.

[1] V. Smaluk, NIMA 888, 22 (2018). [2] B. W. Zotter, CERN-SPS-81-14-DI (1981).

Theories for longitudinal single-bunch impedance effects

- Haissinski equation [1]
  - Exact solution of Vlasov-Fokker-Planck equation below microwave instability threshold.
  - Bottom-up predictions of potential-well lengthening: Impedance calculations→Simulations→Experiments.

$$\lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2}-\frac{I}{\sigma_{z0}}\int_z^\infty dz'\int_{-\infty}^\infty W_\parallel(z'-z'')\lambda_0(z'')dz''} \qquad I = \frac{I_b\sigma_{z0}}{c\eta\sigma_{\delta0}^2(E/e)}$$

- Zotter's equation [2]
  - Simple scaling law of bunch lengthening, widely used to extrapolate effective impedance from bunch length measurements.

$$x^3 - x - \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta0}^2(E/e)}\mathbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1} = 0$$

$x = \sigma_z/\sigma_{z0}$ is the bunch lengthening factor.
$\kappa = \sqrt{2\pi}$ was used in [2].
$\kappa = 4\sqrt{\pi}$ is more consistent to Haissinski equation and experiments [3].

- Connecting Haissinski equation (HE) and Zotter's equation (ZE)
  - HE is self-consistent, knowing $Z_\parallel(k)$ means "knowing everything", except for $Z_\parallel/n$.
  - ZE is easy to use, but it has certain applicability conditions.
  - In this talk, we examine ZE and present a new equation for potential-well bunch lengthening.

[1] J. Haissinski, Nuovo Cimento 18, 1 (1973). [2] B. W. Zotter, CERN-SPS-81-14-DI (1981). [3] V. Smaluk, NIMA 888, 22 (2018).

A revisit of Zotter's equation

- How Zotter derived the cubic equation?
  - Equations of motion + Incoherent tune shift (from effective impedance) + Energy spread condition [1]

$$\frac{d^2z}{ds^2} = -\frac{\omega_{z0}^2}{c^2}z + \eta F(z,s) \qquad F(z,s) = I_n\int_{-\infty}^{\infty} W_{\|}(z-z')\lambda(z',s)dz' = \frac{cI_n}{2\pi}\int_{-\infty}^{\infty} dkZ_{\|}(k)\tilde{\lambda}(k,s)e^{ikz}$$

$$\omega_z^2 = \omega_{z0}^2\left(1-\xi Z_1\right) \qquad \sigma_z\omega_z = \sigma_{z0}\omega_{z0} = -c\eta\sigma_{\delta 0} \qquad Z_1(\sigma_z) = -\frac{\sqrt{2\pi}c^3}{\omega_0^3\sigma_z^3}\textbf{Im}\left(\frac{Z_{\|}}{n}\right)_{eff}^{m=1} \qquad \left(\frac{Z_{\|}}{n}\right)^m_{\textbf{eff}} = \frac{\sum_{p=-\infty}^{\infty}\frac{Z_{\|}(\omega_{mp})}{p}h_m(\omega_{mp})}{\sum_{p=-\infty}^{\infty}h_m(\omega_{mp})}$$

Incorrect factor leading to $\kappa = \sqrt{2\pi}$

Effective impedance

- An alternative way to derive the cubic equation
  - Equations of motion + Incoherent tune shift (from from linearized wake force) + Energy spread condition
  - The standard formulae of effective impedance is unnecessary

$$\frac{d^2z}{ds^2} = -\frac{\omega_{z0}^2}{c^2}z + \eta F(z,s) \qquad F(z,s) \approx F_1(s)z \qquad F_1 = \int_{-\infty}^{\infty} dz\lambda(z,s)\frac{\partial F(z,s)}{\partial z} = \frac{icI_n}{2\pi}\int_{-\infty}^{\infty} dkkZ_{\|}(k)\lambda(k,s)\lambda^*(k,s)$$

Linearization

Average

Linear part of wake force

Effective inductance

$$\frac{d^2z}{ds^2} = -\frac{\omega_z^2}{c^2}z \qquad \frac{\omega_z^2}{c^2} = \frac{\omega_{z0}^2}{c^2} - \frac{c^2\eta I_n L_{eff}}{4\sqrt{\pi}\sigma_z^3}$$

Correct factor

$$\rightarrow \quad x^3 - x - \frac{cI_b}{\kappa\eta\sigma_{z0}\sigma_{\delta 0}^2(E/e)}L_{eff} = 0$$

Modified Zotter's equation

[1] B. W. Zotter, CERN-SPS-81-14-DI (1981).

A new equation derived from Hassinski equation

- rms bunch length σ_z

Definition: $\sigma_z^2 = \int_{-\infty}^{\infty} (z - z_c)^2 \lambda_0(z) dz = \int_{-\infty}^{\infty} z^2 \lambda_0(z) dz - z_c^2$

Wake potential: $\mathbb{W}_{\|}(z) = \int_{-\infty}^{\infty} W_{\|}(z - z')\lambda_0(z')dz'$

$$\frac{d\lambda_0(z)}{dz} + \left[\frac{z}{\sigma_{z0}^2} - \frac{1}{\eta\sigma_\delta^2}F_0(z)\right]\lambda_0(z) = 0 \quad \rightarrow \quad x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}}Z_{\|}^{eff}(x) = 0$$

Trick: Multiply by z and then integrate this equation over z

1) Exact prediction by Haissinski equation
2) A generalized version of Zotter's equation

Normalized current:

$$I = \frac{I_b\sigma_{z0}}{c\eta\sigma_{\delta0}^2(E/e)}$$

$$x = \sigma_z/\sigma_{z0}$$

"Effective impedance" for bunch lengthening:

$$Z_{\|}^{eff} = \frac{2\pi}{c}\int_{-\infty}^{\infty} dz(z - z_c)\lambda_0(z)\mathbb{W}_{\|}(z)$$

Stretching force average over charge density

$$Z_{\|}^{eff} = -\int_{-\infty}^{\infty} dk Z_{\|}(k)\tilde{\lambda}_0(k)\left[i\frac{d}{dk}\tilde{\lambda}_0^*(k) + z_c\tilde{\lambda}_0^*(k)\right]$$

Both real and imaginary parts of impedance contribute to bunch lengthening if the bunch is deformed

- σ_z is sensitive to imaginary part of impedance
- If real part of impedance is large, it also contributes to bunch lengthening

A new equation derived from Hassinski equation

- Relation between the "quadratic" equation and Zotter's equation

$$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}} Z_\parallel^{eff}(x) = 0 \qquad Z_\parallel^{eff} = -\int_{-\infty}^{\infty} dk Z_\parallel(k)\tilde{\lambda}_0(k)\left[i\frac{d}{dk}\tilde{\lambda}_0^*(k) + z_c\tilde{\lambda}_0^*(k)\right]$$

  - First, we assume a Gaussian bunch to approximate the Hassinski distribution.

$$z_c = \frac{I\sigma_{z0}c}{\pi}\int_0^\infty dk\mathbf{Re}[Z_\parallel(k)]e^{-k^2\sigma_z^2} \qquad Z_\parallel^{eff} = -2\sigma_z^2\int_0^\infty dkk\mathbf{Im}[Z_\parallel(k)]e^{-k^2\sigma_z^2}$$

1) $Z_\parallel^{eff}$ and z_c depends on the imaginary and real parts of impedance, respectively.

2) For a full understanding of broad-band impedance, we need to measure both z_c (real part) and $Z_\parallel^{eff}$ (imaginary part).

  - Second, we approximate the ring impedance by $Z_\parallel(k) = -ikcL_{eff}$. Immediately, we obtain Zotter's equation.

$$Z_\parallel^{eff} \propto \frac{1}{x} \longrightarrow x^3 - x - \frac{cI_b}{\kappa\eta\sigma_{z0}\sigma_{\delta 0}^2(E/e)}L_{eff} = 0$$

$$L_{eff} = -\frac{1}{\omega_0}\mathbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1} = \frac{2\sigma_{z0}}{\sqrt{\pi}c}Z_\parallel^{eff}$$

L_{eff}, $(Z_\parallel/n)_{eff}^{m=1}$, and $Z_\parallel^{eff}$ should be evaluated at nominal bunch length σ_{z0}

  - We conclude that Zotter's equation is one special case of self-consistent "quadratic" equation. ZE is only a good approximation for electron storage rings where the inductance is the dominant impedance.

Potential-well bunch lengthening for various impedance sources

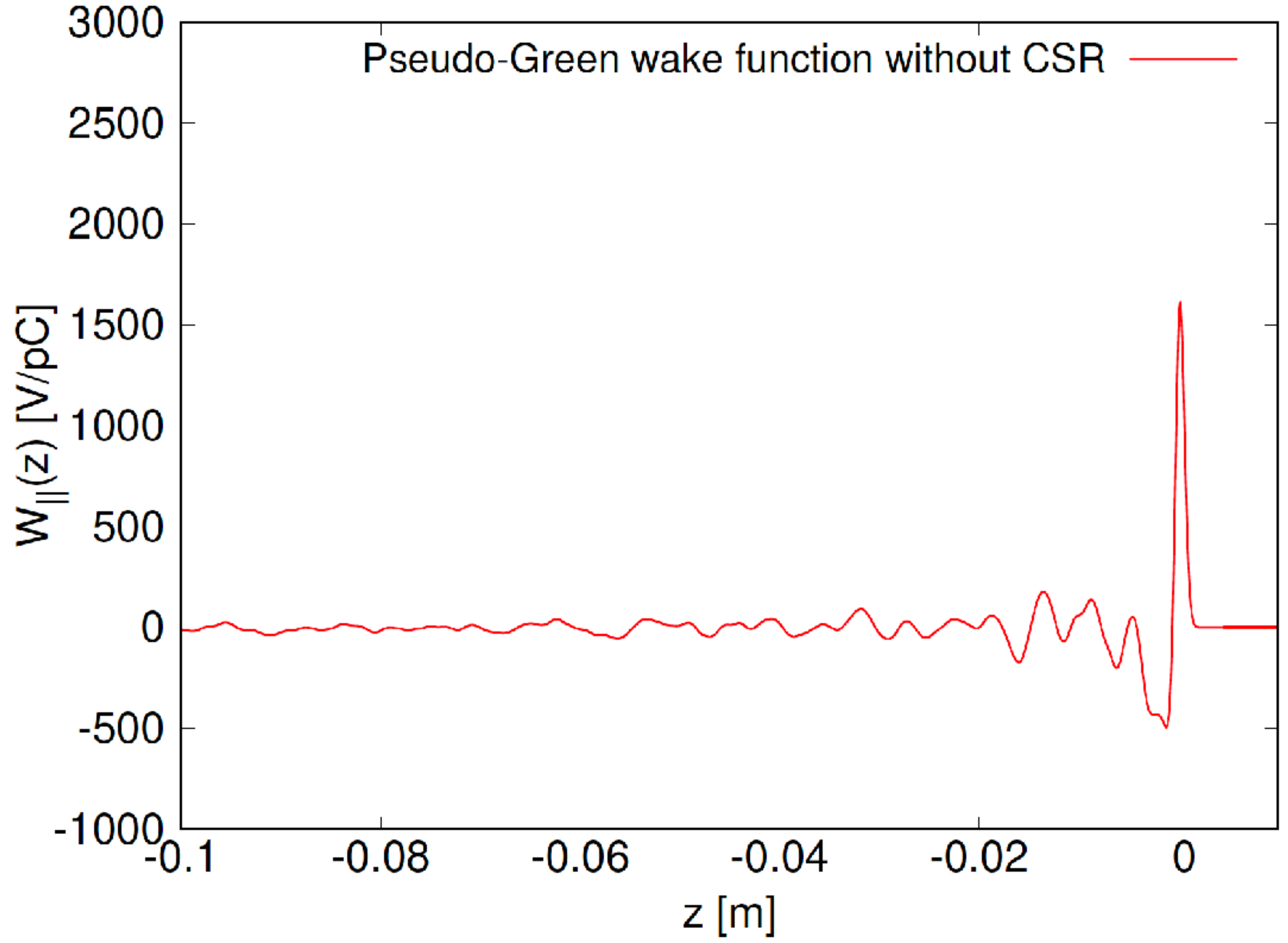
- A table of z_c and $Z_{\parallel}^{eff}$ with Gaussian bunch approximation [1]
  - Bunch shortening for positive momentum compaction: Pure capacitance, Free-space steady-state CSR and CWR

[1] D. Zhou, G. Mitsuka and T. Ishibashi, arXiv:2307.01286 (2023).

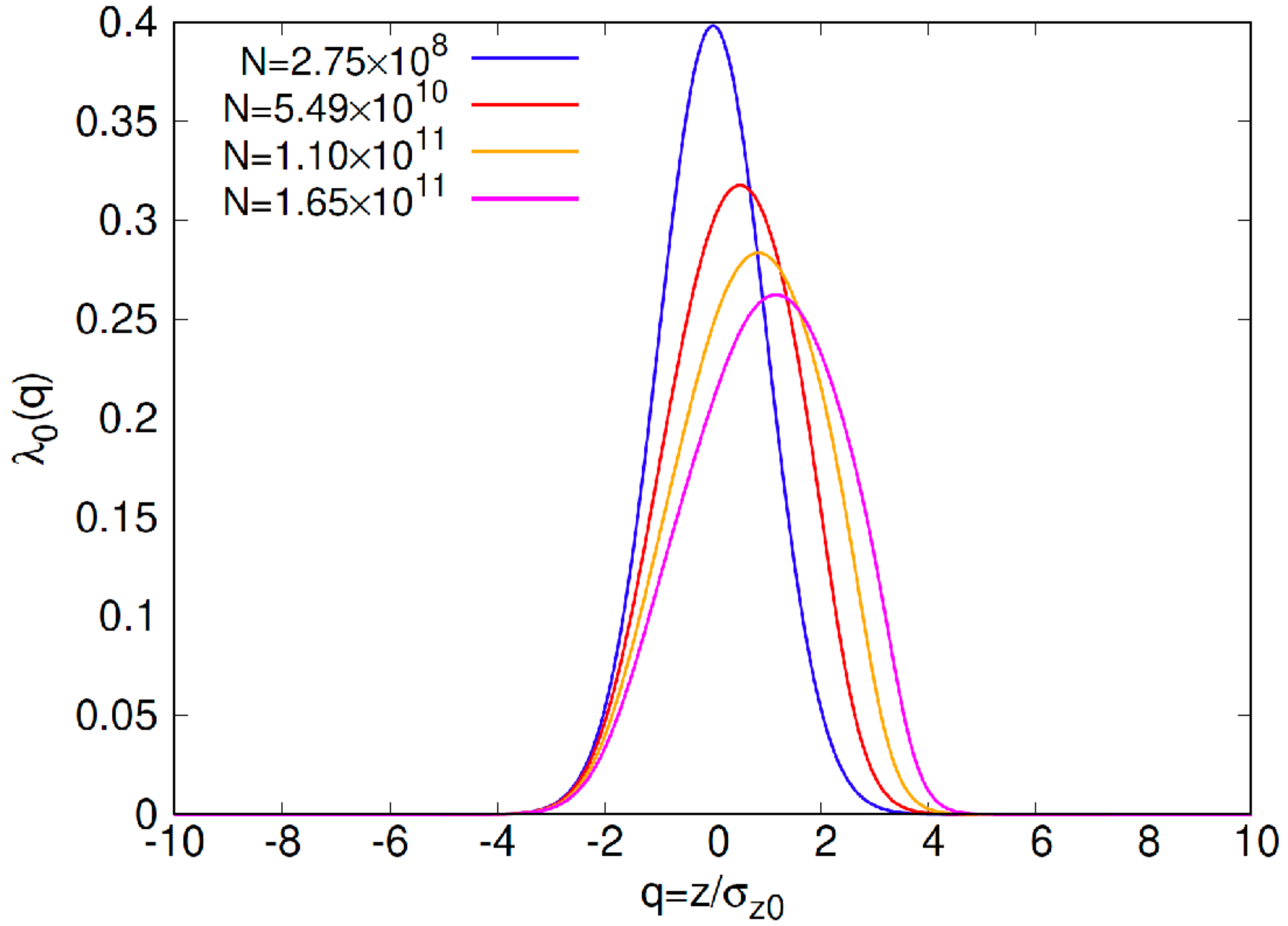
Description	Impedances $Z_{\parallel}(k)$	Effective impedance $Z_{\parallel}^{\rm eff}(x)$	Centroid shift $z_c(x)$
Pure inductance	$-ikcL$	$\frac{\sqrt{\pi}cL}{2\sigma_{z0}x}$	0
Pure resistance	R	0	$\frac{IcR}{2\sqrt{\pi}x}$
Pure capacitance	$\frac{i}{kcC}$	$-\frac{\sqrt{\pi}\sigma_{z0}x}{cC}$	0
Resistive wall	$\frac{L}{2\pi b}\left[1-i\mathrm{sgn}[k]\right]\sqrt{\frac{\lvert k\rvert Z_0}{2\sigma_c}}$	$\frac{L}{2\pi b}\sqrt{\frac{Z_0}{2\sigma_c}}\frac{\Gamma\left(\frac{5}{4}\right)}{\sqrt{\sigma_{z0}x}}$	$\frac{L}{2\pi b}\sqrt{\frac{Z_0}{2\sigma_c}}\frac{cI\Gamma\left(\frac{3}{4}\right)}{\pi\sqrt{\sigma_{z0}}x^{3/2}}$
	L: chamber length; b: chamber radius; σ_c: Conductivity.		
Steady-state CSR in free-space	$\frac{Z_0}{3^{1/3}}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\Gamma\left(\frac{2}{3}\right)(k\rho)^{1/3}$	$-\frac{Z_0\Gamma\left(\frac{2}{3}\right)\rho^{1/3}}{2\cdot 3^{1/3}}\frac{\Gamma\left(\frac{7}{6}\right)}{(\sigma_{z0}x)^{1/3}}$	$\frac{3^{1/6}\Gamma^2\left(\frac{2}{3}\right)}{4\pi}\frac{cIZ_0\rho^{1/3}}{\sigma_{z0}^{1/3}x^{4/3}}$
	ρ: bending radius; assume $2\pi\rho$ for the total length of dipoles.		
Steady-state CWR in free-space [18]	$\frac{1}{16}Z_0\theta_0^2Lk\left(1-\frac{2i}{\pi}\ln\frac{k}{k_c}\right)$	$-\frac{Z_0\theta_0^2L}{32\sqrt{\pi}x\sigma_{z0}}\left[Y+2\ln\left(k_cx\sigma_{z0}\right)\right]$	$\frac{cIZ_0\theta_0^2L}{32\pi\sigma_{z0}x^2}$
	L: wiggler length; θ_0: wiggler deflection angle; k_c: fundamental frequency of wiggler radiation; $Y=-2+\gamma_E+\ln 4\approx -0.0365$.		
Resonator model ($Q>1/2$)	$\frac{R_s}{1+iQ(k_r/k-k/k_r)}$	$-\frac{\pi R_s\sigma_z}{Q'}\left[\frac{k_rQ'}{\sqrt{\pi}Q}+\sigma_z\mathrm{Im}[k_1^2w(k_1\sigma_z)]\right]$	$\frac{cI\sigma_{z0}R_s}{2Q'}\mathrm{Re}[k_1w(k_1\sigma_z)]$
	R_s: shunt impedance; Q: quality factor; k_r: resonant frequency; $Q'=\sqrt{Q^2-1/4}$; $k_1=\frac{k_r}{Q}[-i/2+Q']$; $w(z)=e^{-z^2}[1-i\mathrm{Erfi}(z)]$; Erfi$(z)$: imaginary error function; $\sigma_z=\sigma_{z0}x$.		

Example 1: SuperKEKB LER

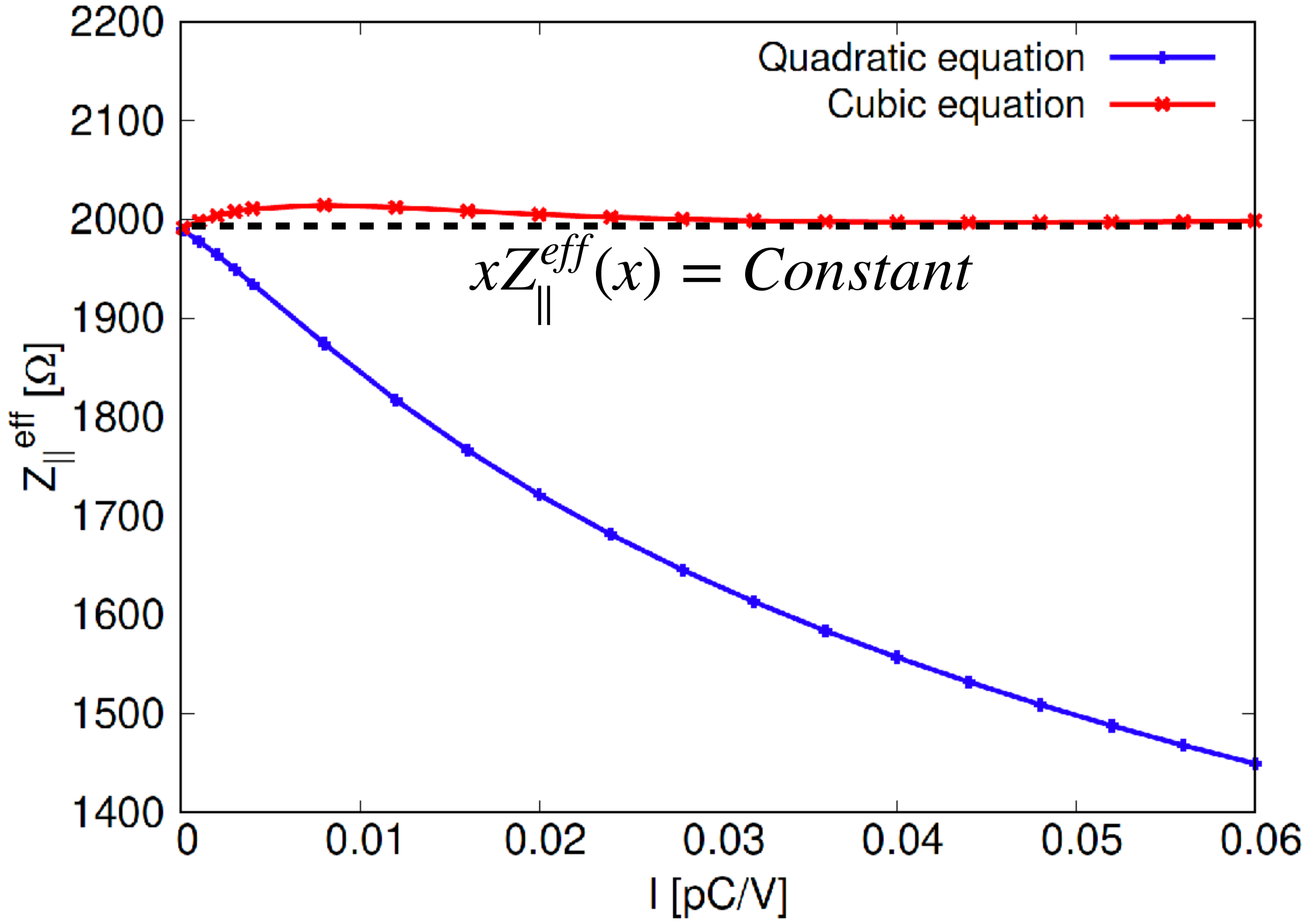
$$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}} Z_{\parallel}^{eff}(x) = 0$$

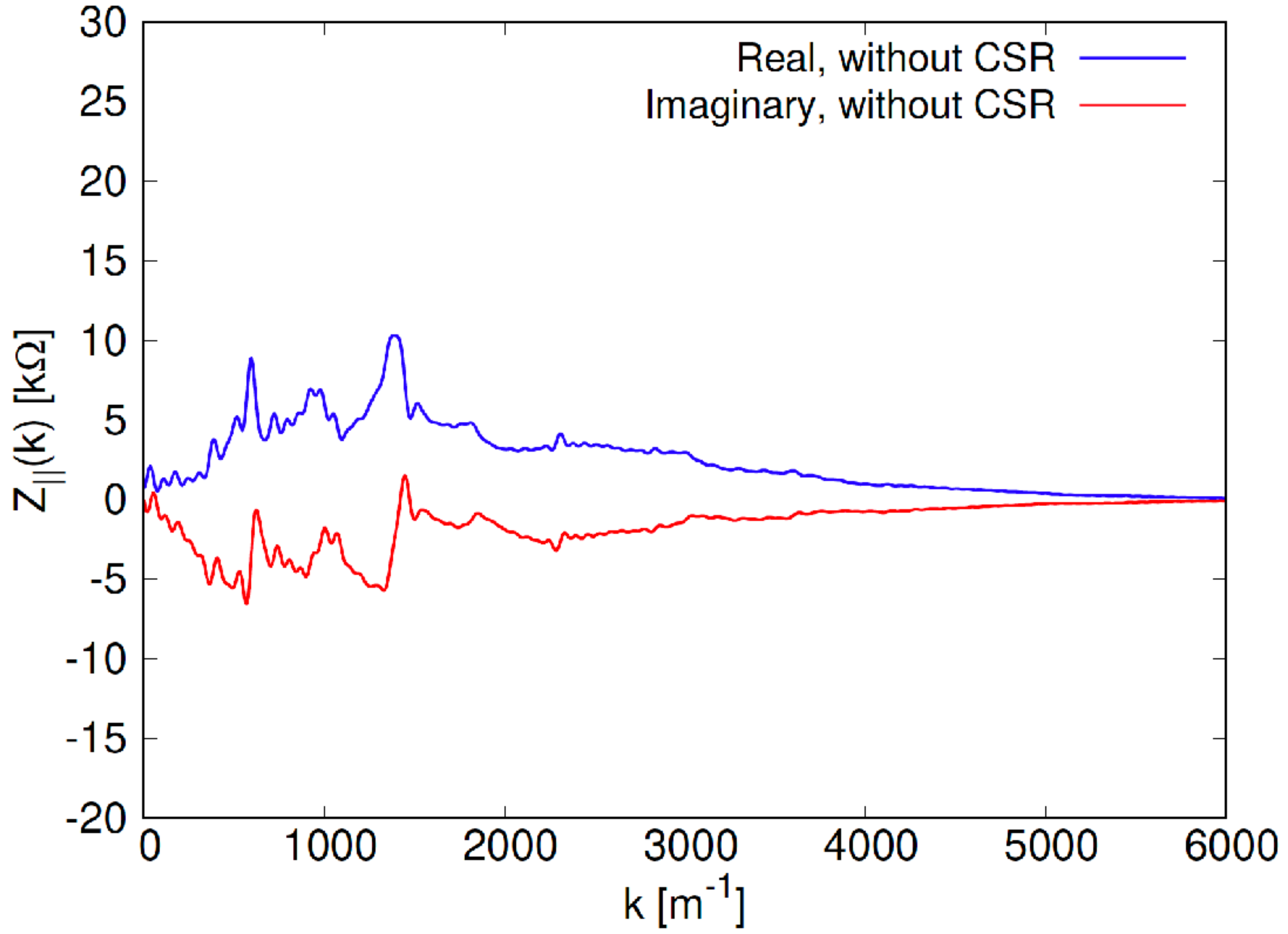


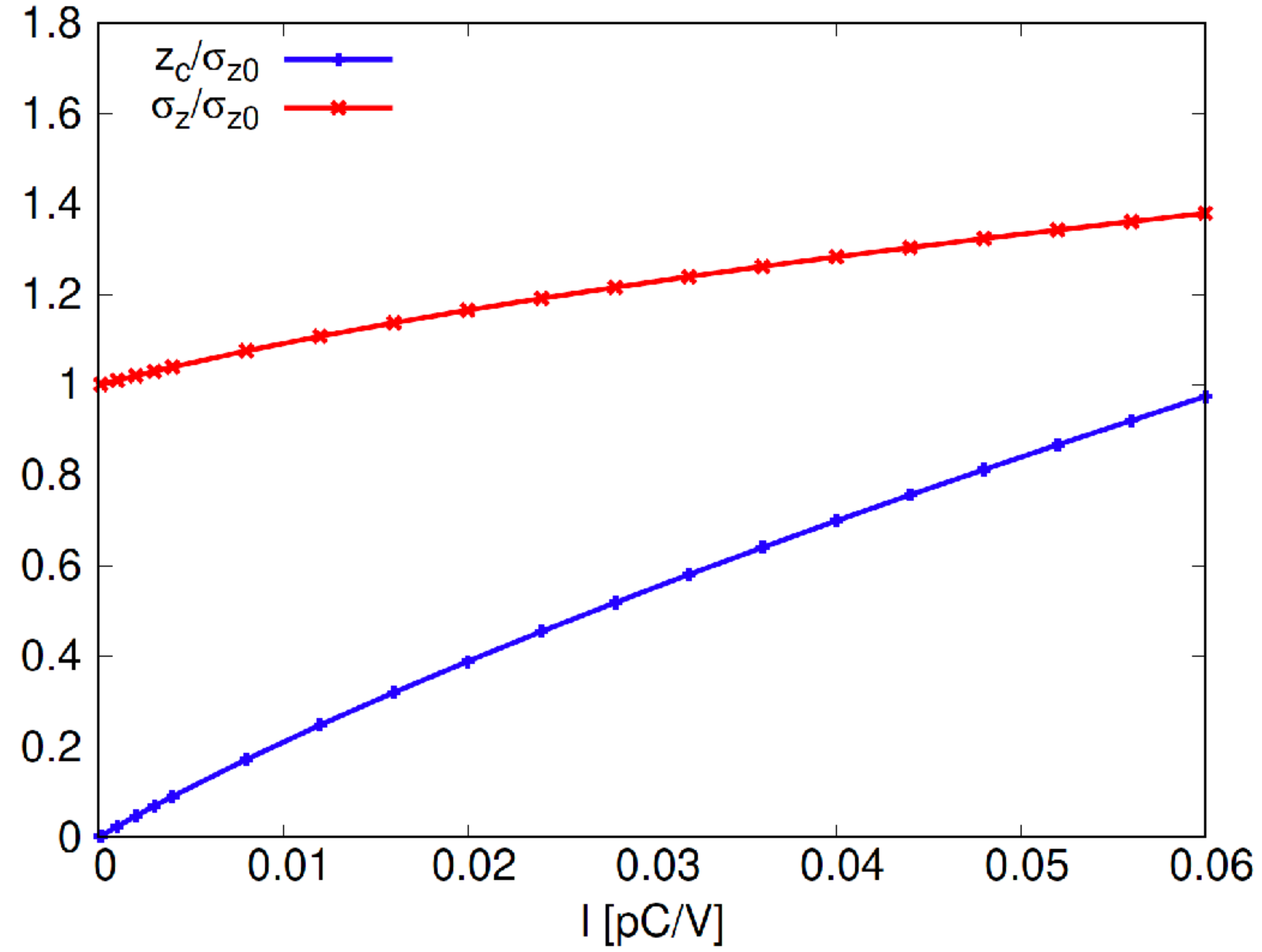
Haissinski distribution



Effective impedance





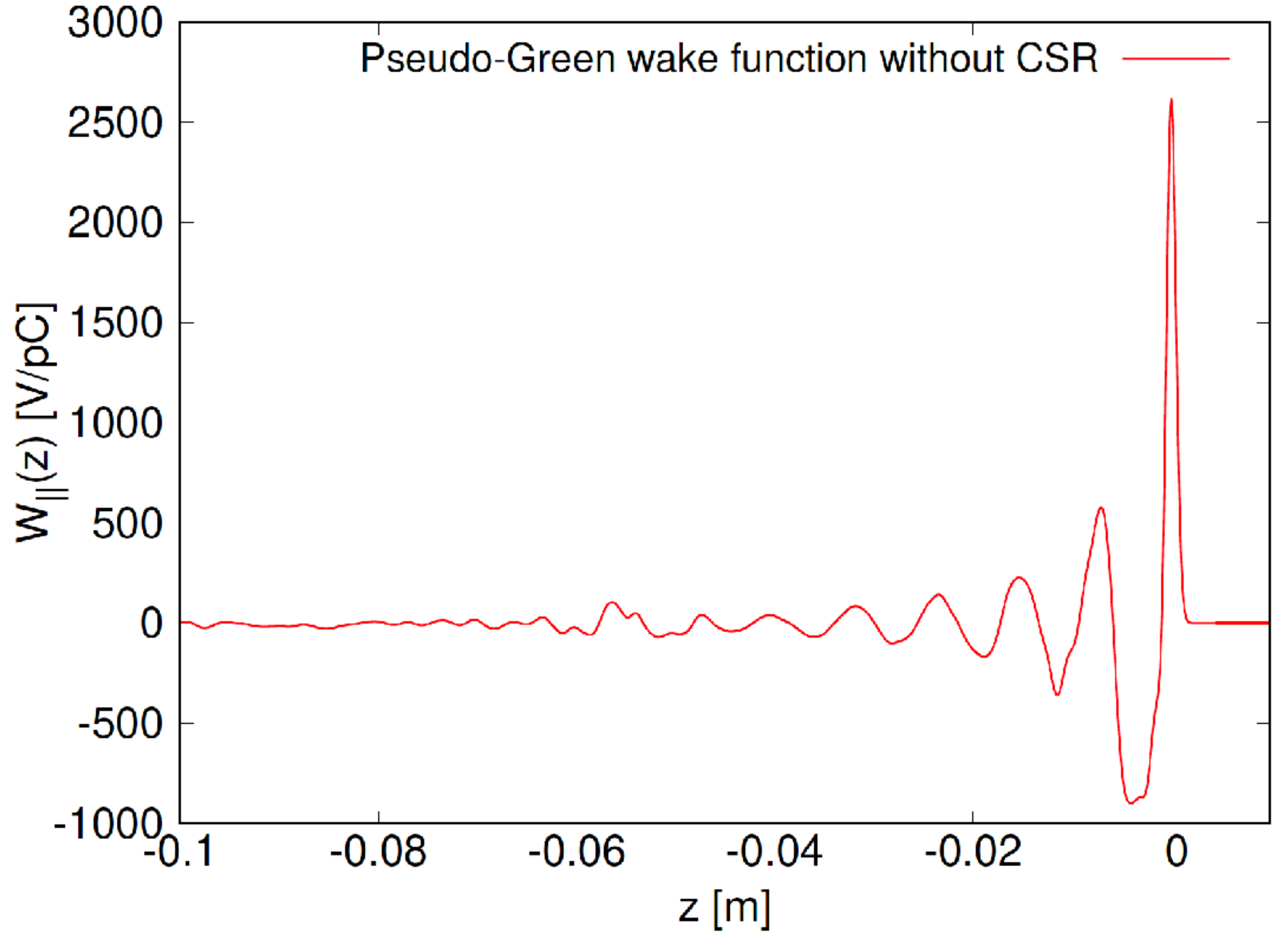


component	k_z [V/pC]	R [Ω]	L [nH]
ARES cavities	9.5	671.9	-
resistive-wall	3.0	213.1	9.1
flanges (φ150, HELICOFLEX)	1.0	70.0	-0.7
MO-flanges	0.0	1.4	5.2
welding-gaps	0.0	0.3	1.4
comb-type bellows	0.9	66.3	5.3
longitudinal feedback kicker	0.8	57.6	-0.8
transverse feedback kicker	0.4	26.1	0.0
clearing electrodes [4]	0.0	1.7	2.4
vertical collimators	0.1	8.2	5.9
horizontal collimators	0.3	17.6	5.6
tapered beam-pipes	0.9	61.0	1.4
QCS beam-pipes	0.1	5.1	0.6
others	1.9	137.3	3.2
Total	18.9	1337.6	30.2

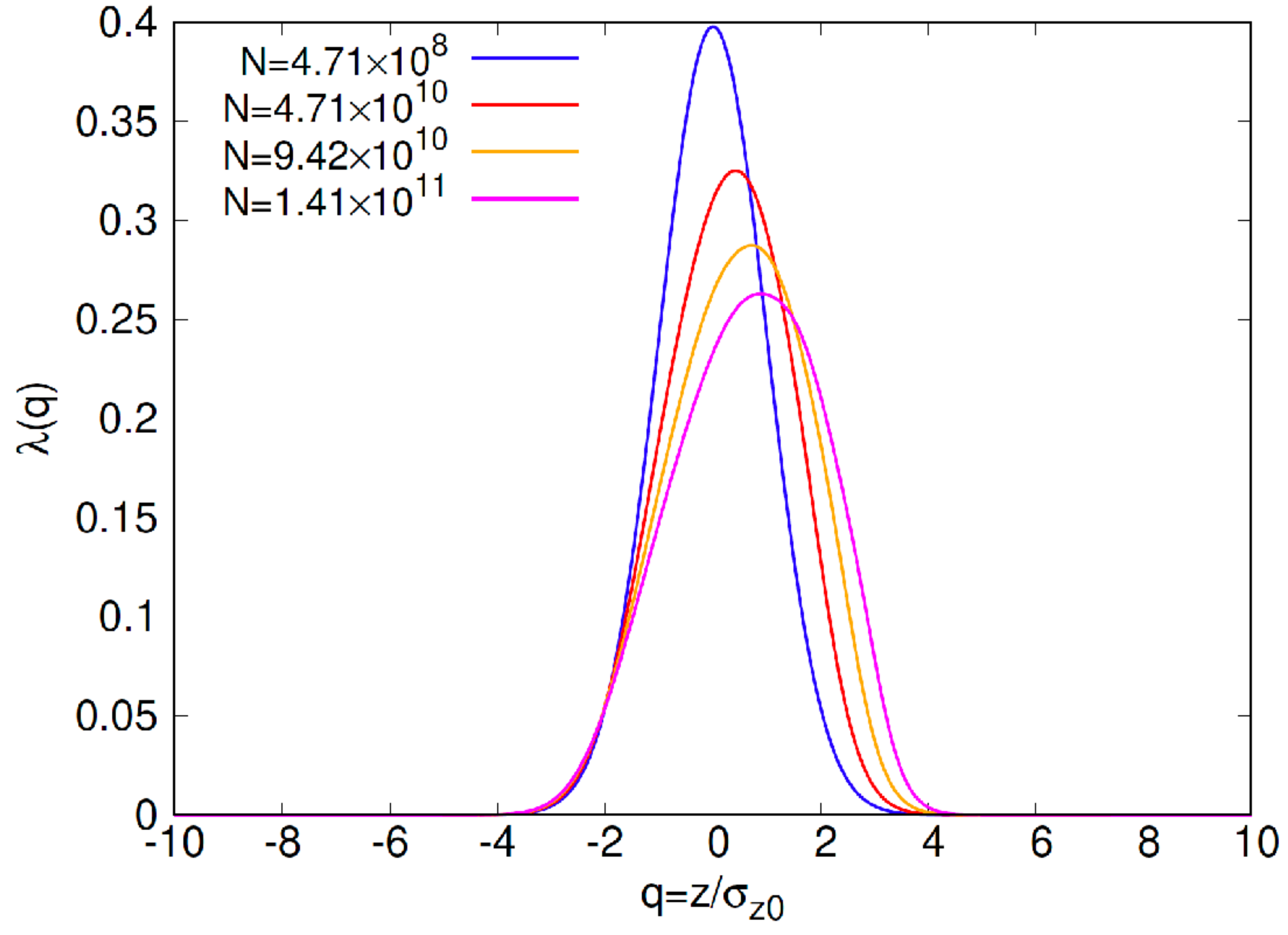
T. Ishibashi et al., ICFA Newsletter #85, 2023

Example 2: SuperKEKB HER

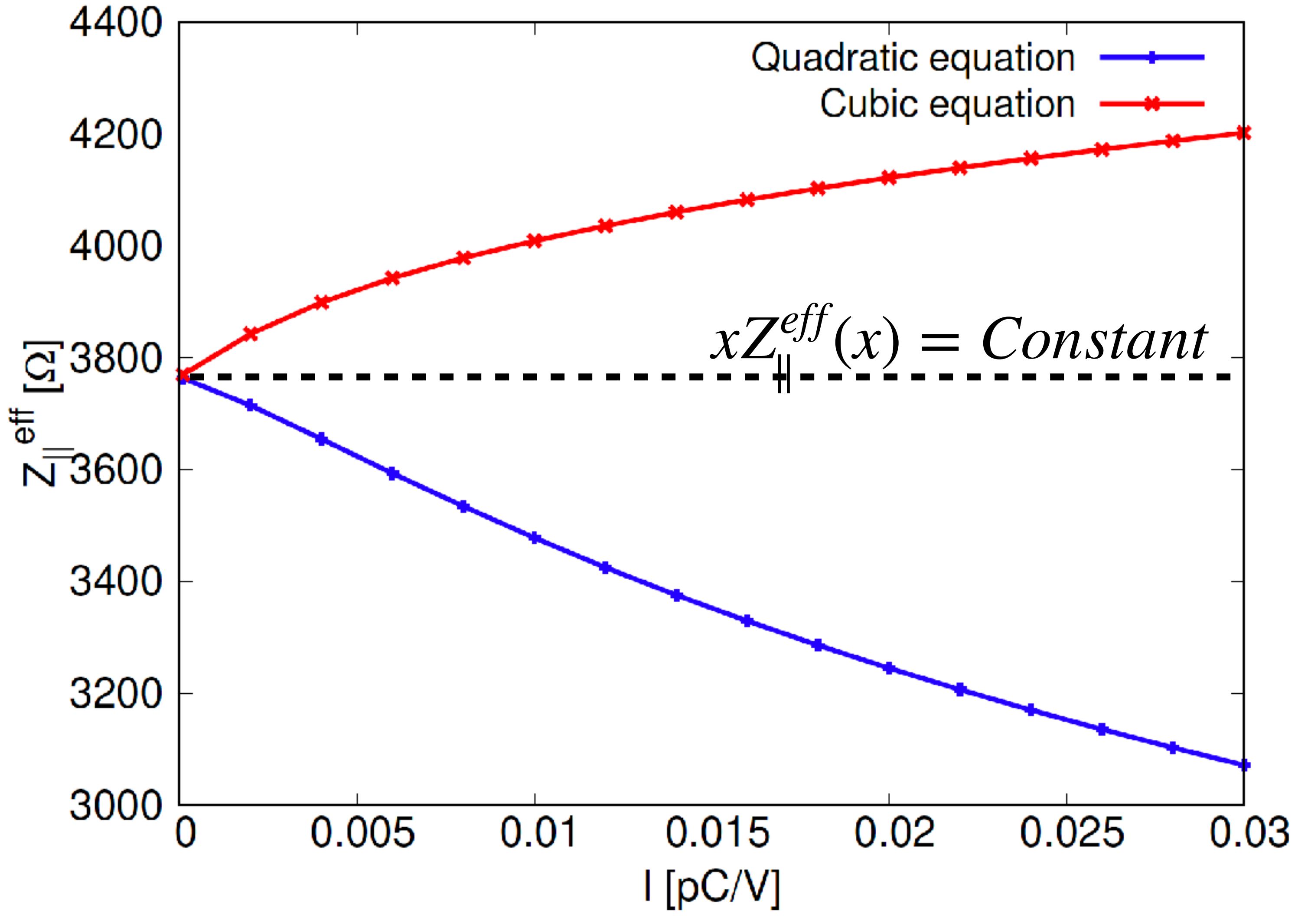
$$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}} Z_{\parallel}^{eff}(x) = 0$$

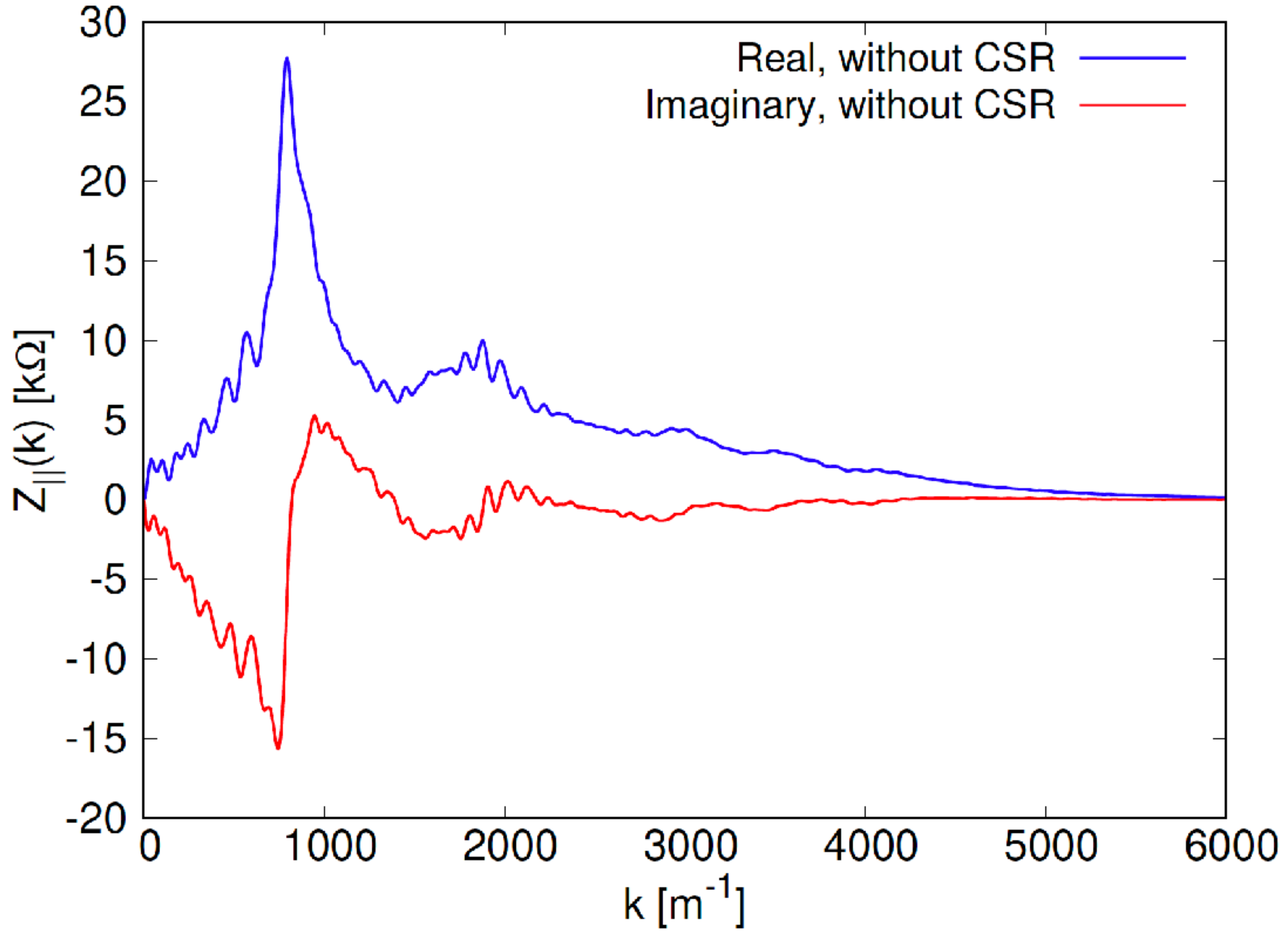


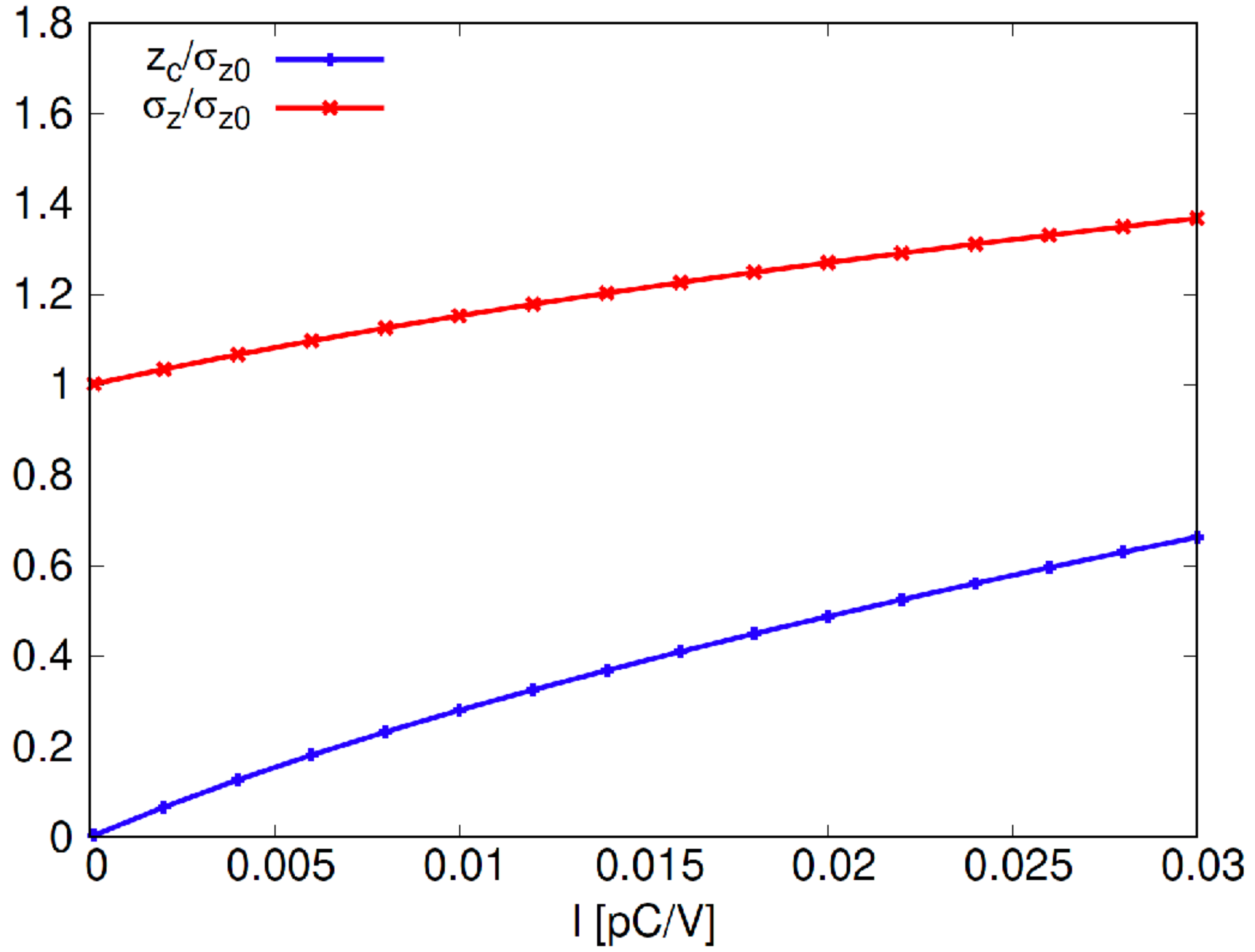
Haissinski distribution



Effective impedance



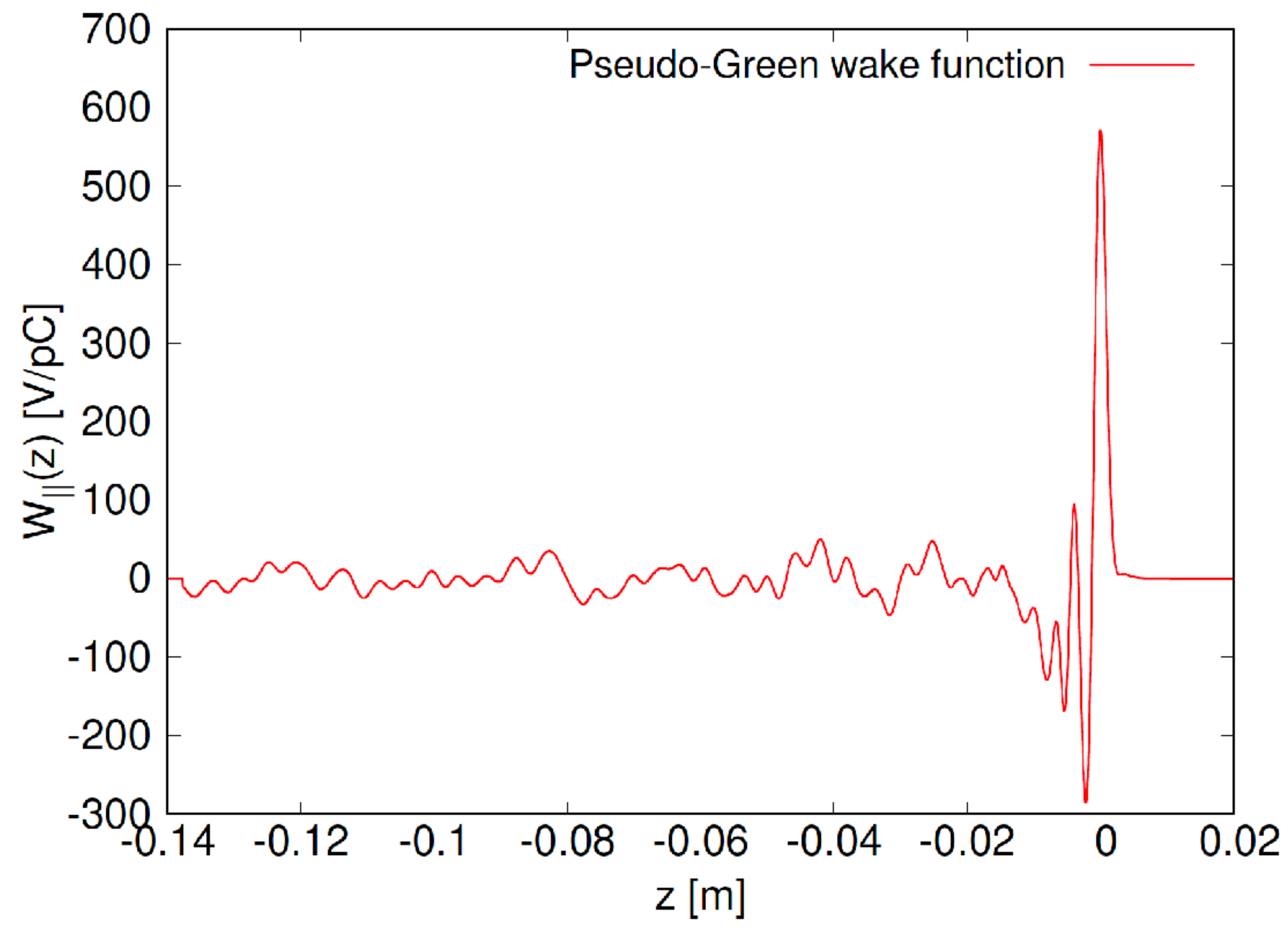




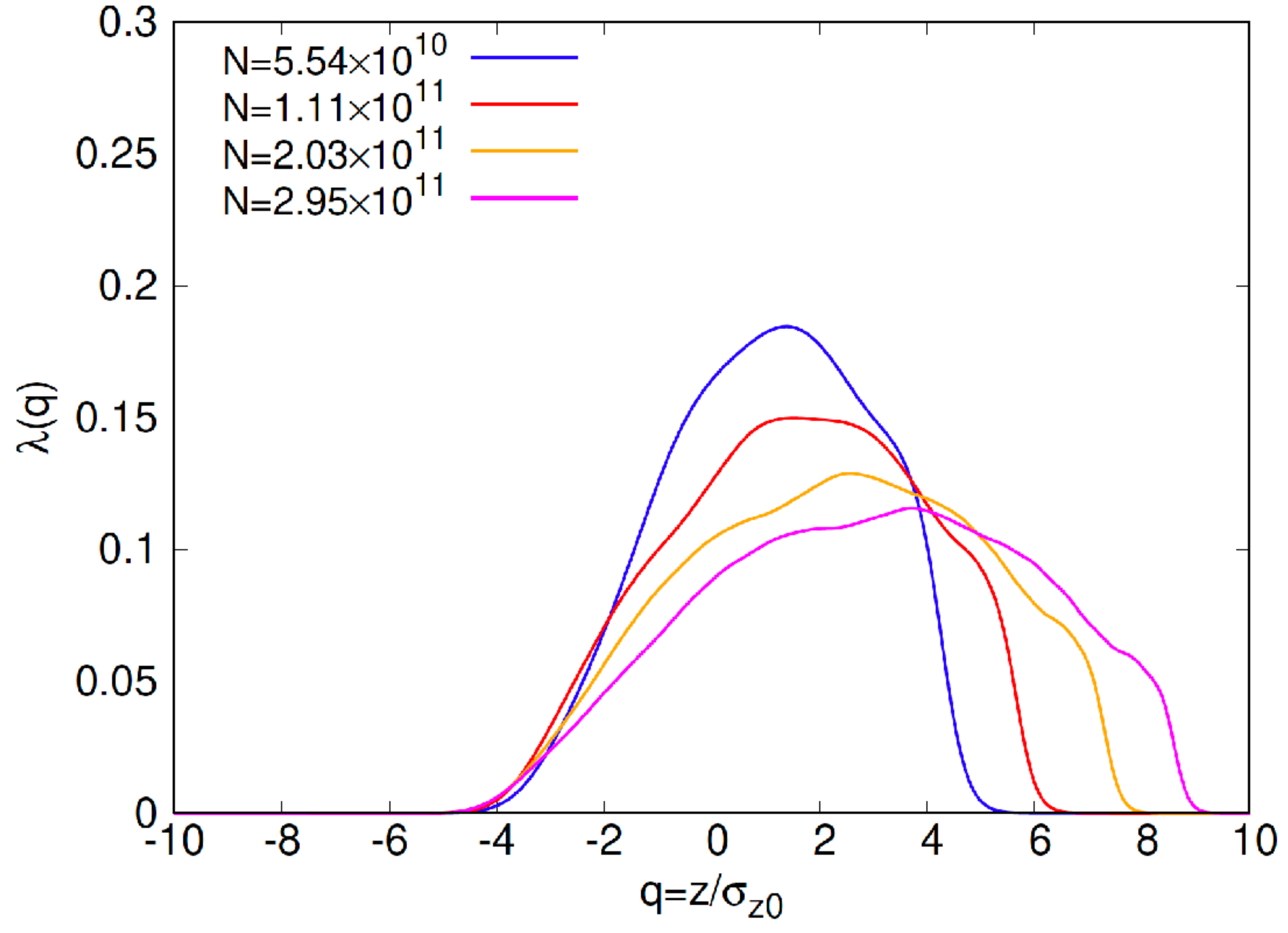
component	k_z [V/pC]	R [Ω]	L [nH]
Superconducting cavity	14.3	845.1	-
ARES cavities	3.8	225.0	-
resistive-wall	4.9	289.5	7.4
flanges (φ150, HELICOFLEX)	2.4	142.6	36.4
flanges (race-track, HELICOFLEX)	1.0	60.5	-0.3
MO-flange	0.0	0.5	0.8
welding-gaps	0.0	0.8	1.6
comb-type bellows	0.2	12.9	0.7
contact-finger-type bellows	4.0	238.1	13.0
transverse feedback kicker	0.4	24.5	0.0
BPM	1.2	71.5	1.5
vertical collimators	1.3	76.2	4.0
horizontal collimators	2.6	150.6	7.2
QCS beam-pipes	0.1	6.8	0.5
pumping-screen	0.3	15.8	3.0
others	0.1	5.6	3.0
Total	36.6	2166.0	70.42

T. Ishibashi et al., ICFA Newsletter #85, 2023

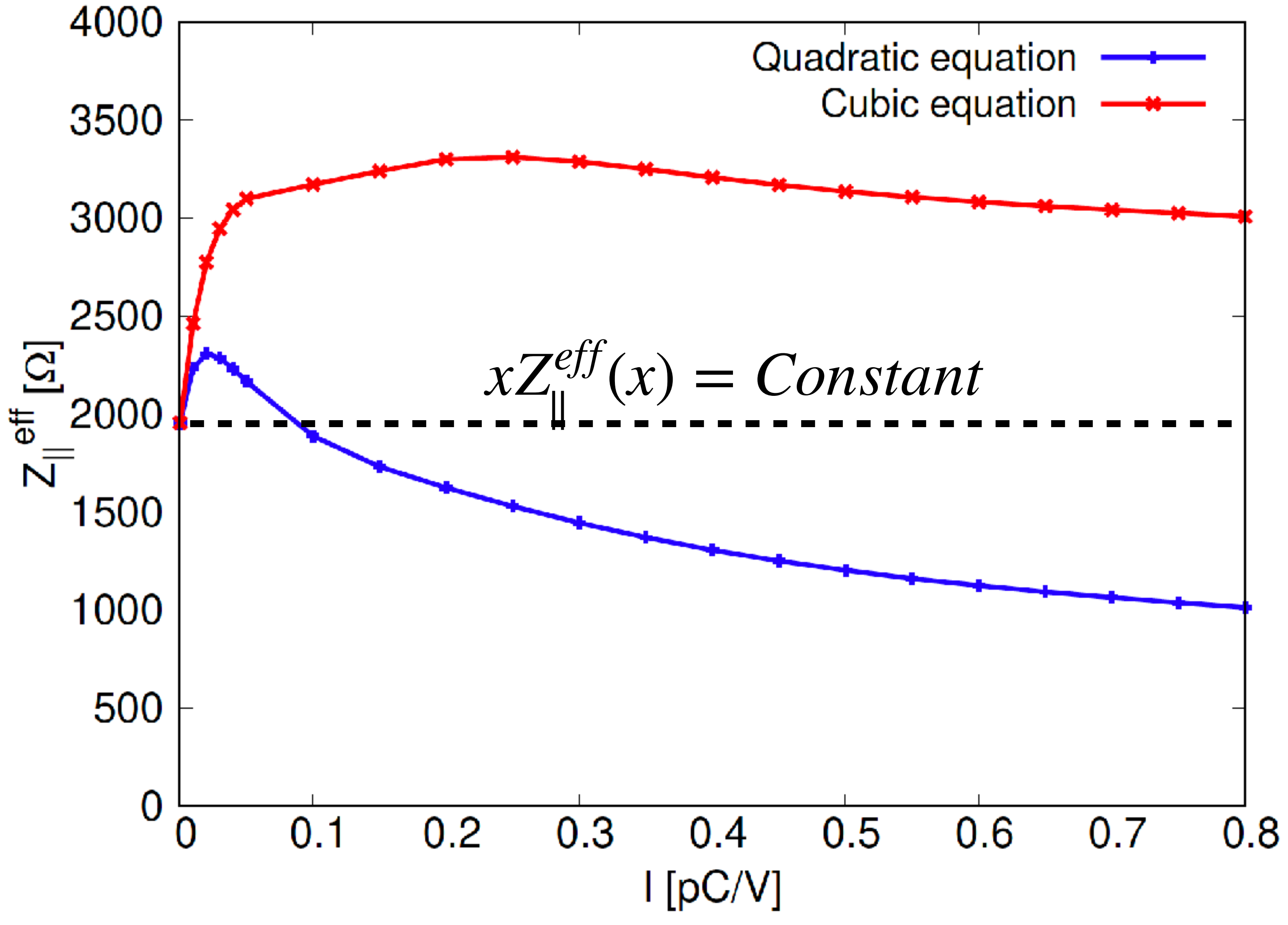
Example 3: SLC damping ring with original vacuum chamber [1]

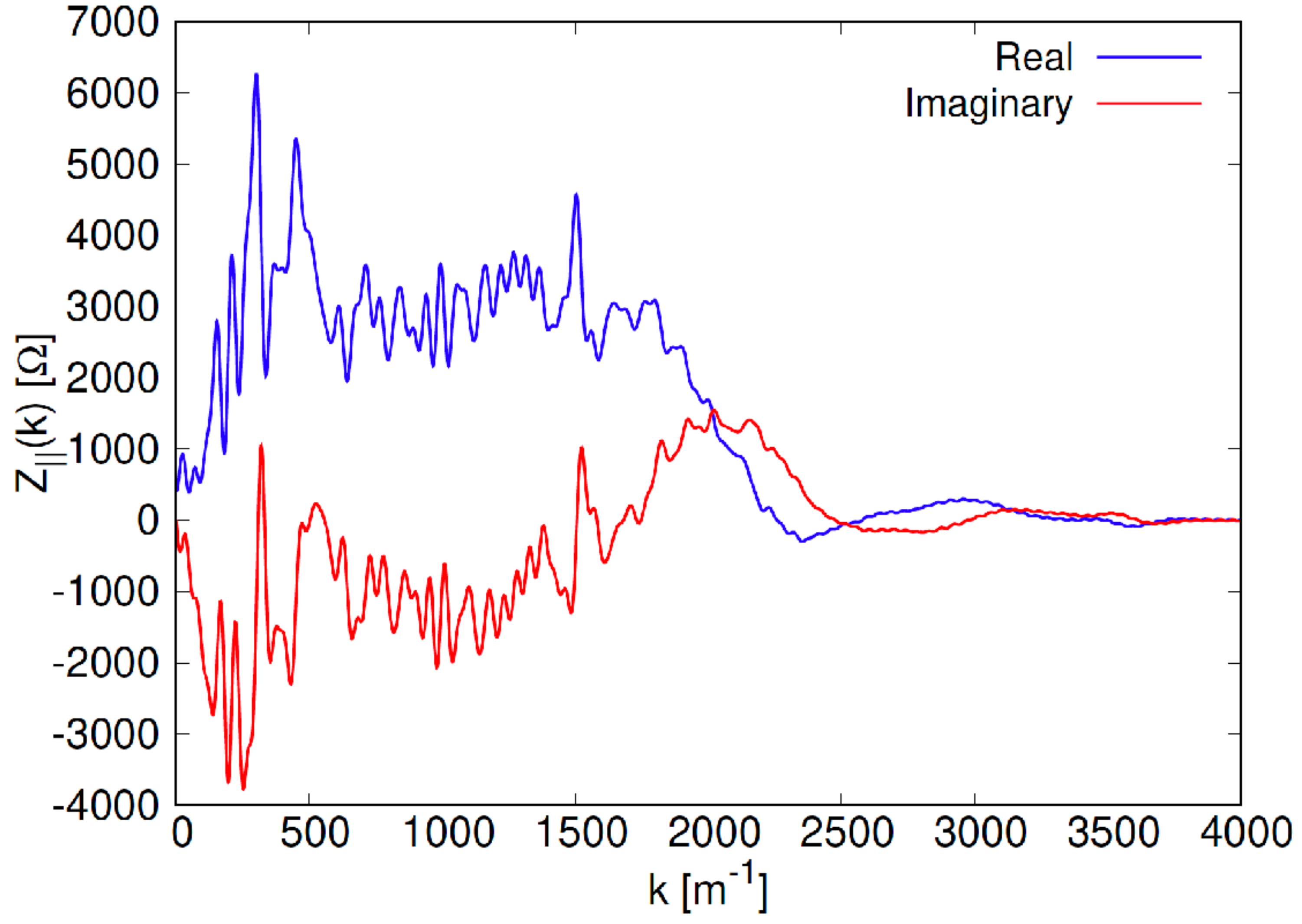


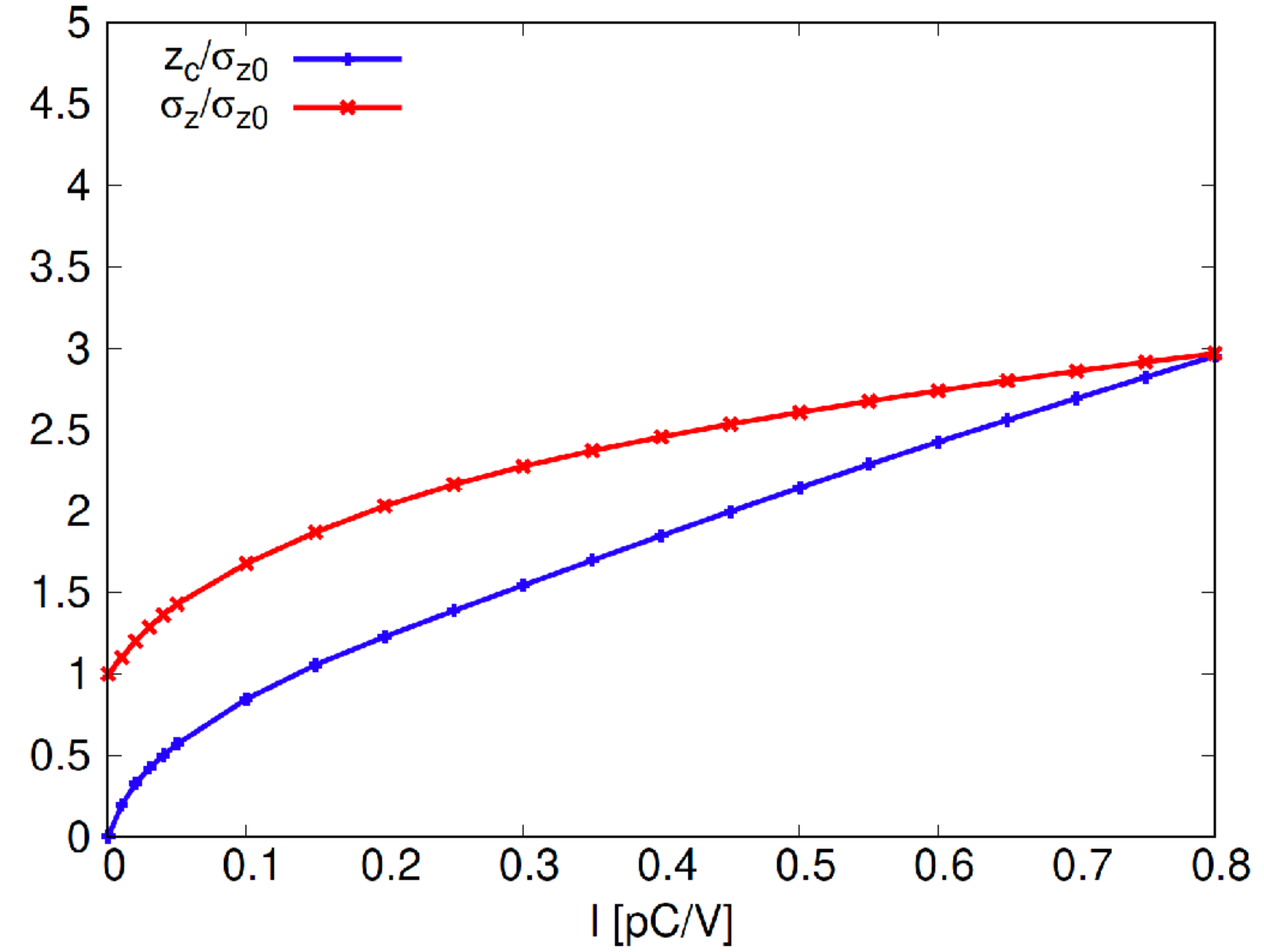
Haissinski distribution



Effective impedance



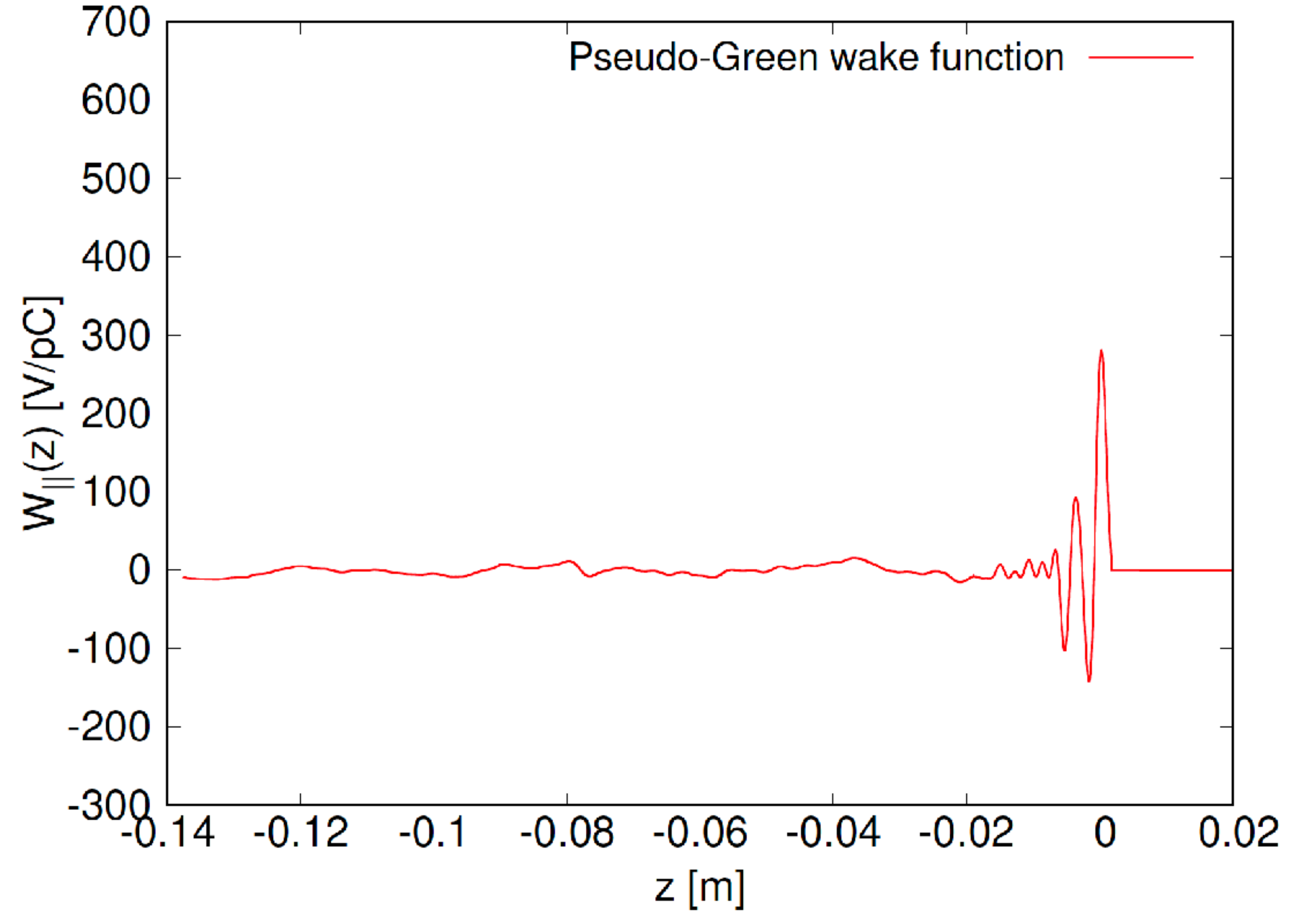




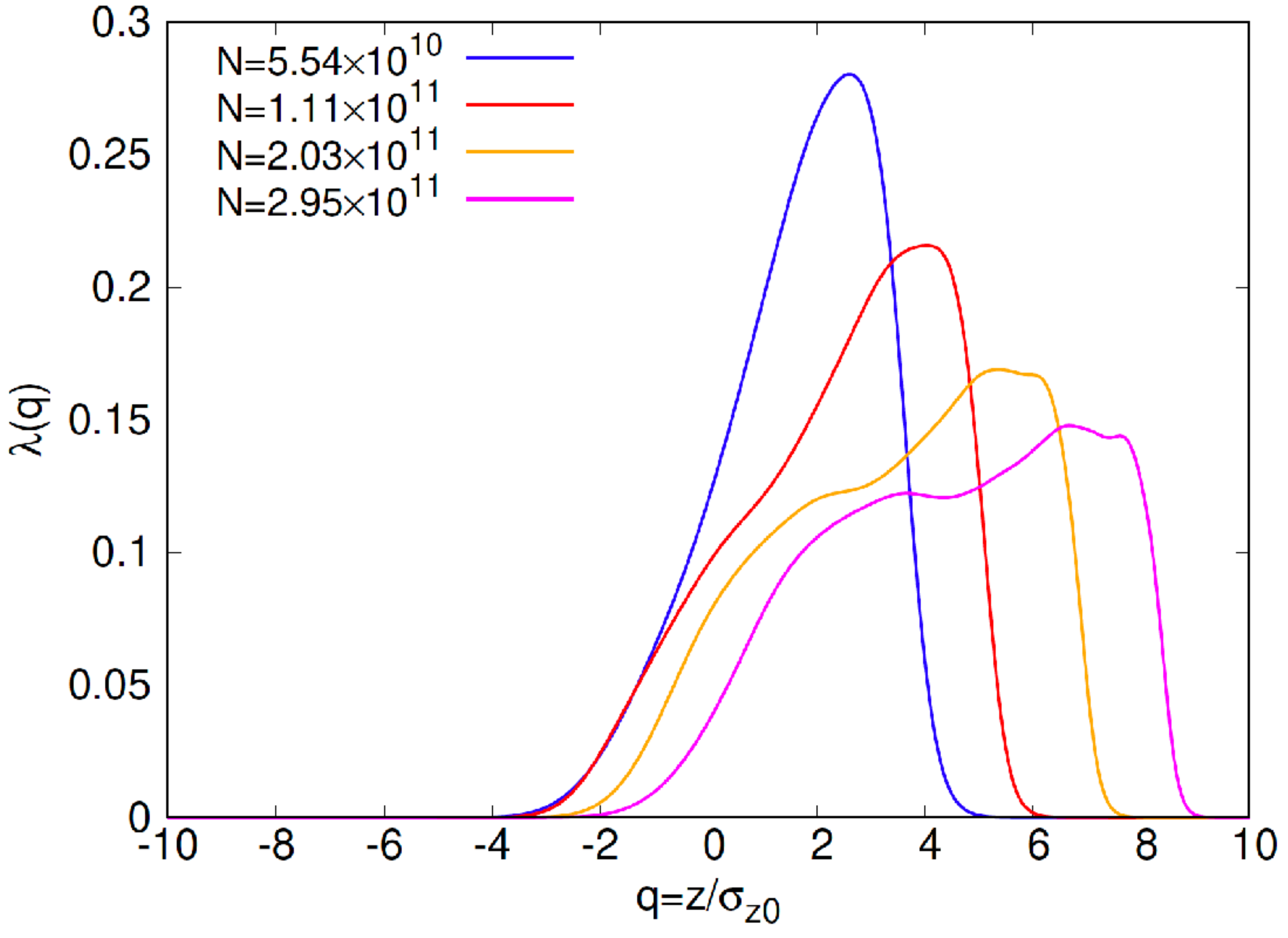
SLC is a special case:
* With original vacuum chamber, inductive character is dominant.
* Zotter's equation does not apply because of large resistive impedance

[1] R. Warnock and K. Bane, Phys. Rev. Accel. Beams 21, 124401 (2018).

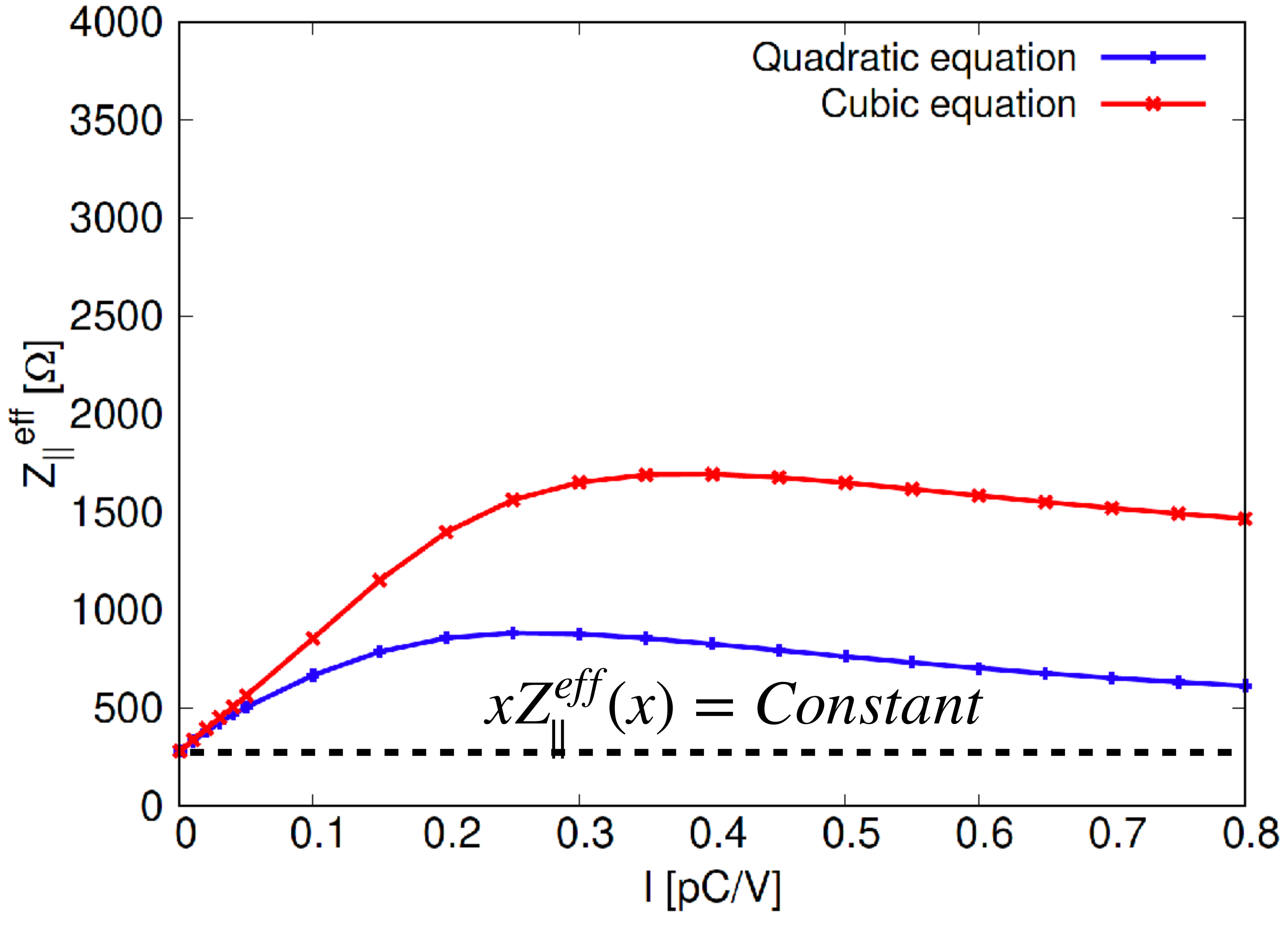
Example 4: SLC damping ring with improved vacuum chamber [1]

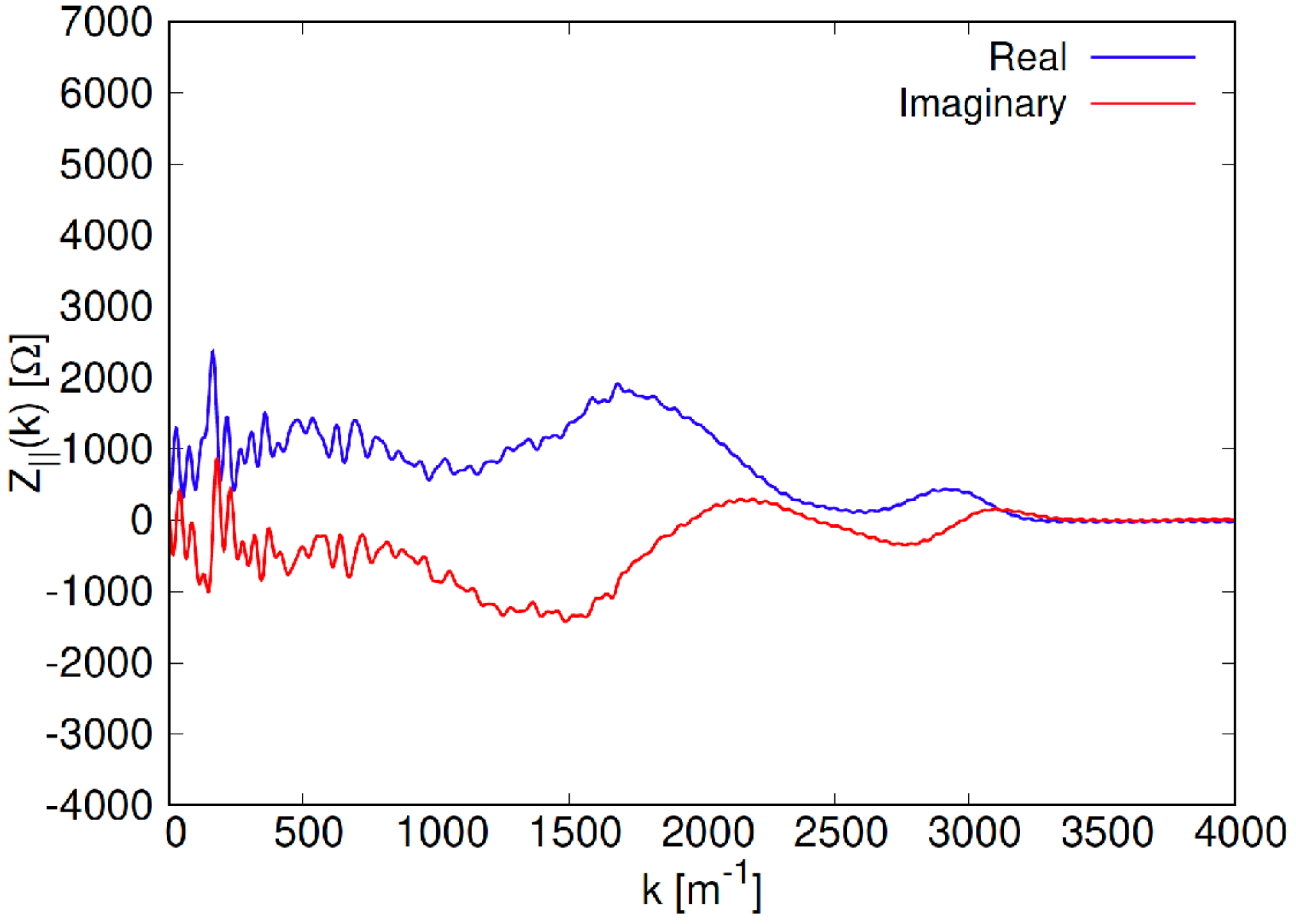


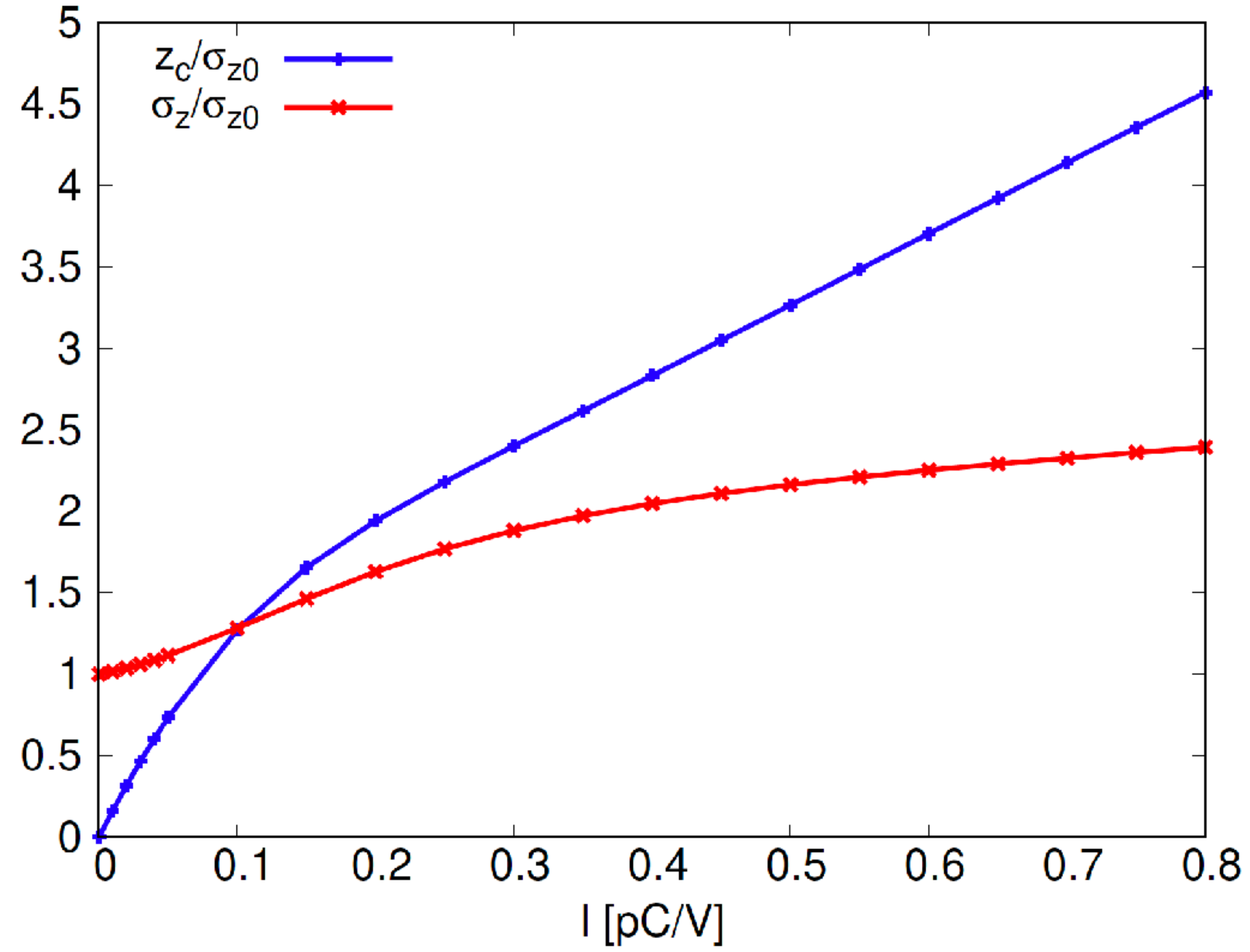
Haissinski distribution



Effective impedance







SLC is a special case:
* With improved vacuum chamber, resistive character is dominant.
* Zotter's equation does not apply because of relative large resistive impedance

[1] R. Warnock and K. Bane, Phys. Rev. Accel. Beams 21, 124401 (2018).

Summary

VFP equation: $$\frac{\partial\psi}{\partial s} + \frac{dz}{ds}\frac{\partial\psi}{\partial z} + \frac{d\delta}{ds}\frac{\partial\psi}{\partial\delta} = \frac{2}{ct_d}\frac{\partial}{\partial\delta}\left[\delta\psi + \sigma_{\delta 0}^2\frac{\partial\psi}{\partial\delta}\right]$$

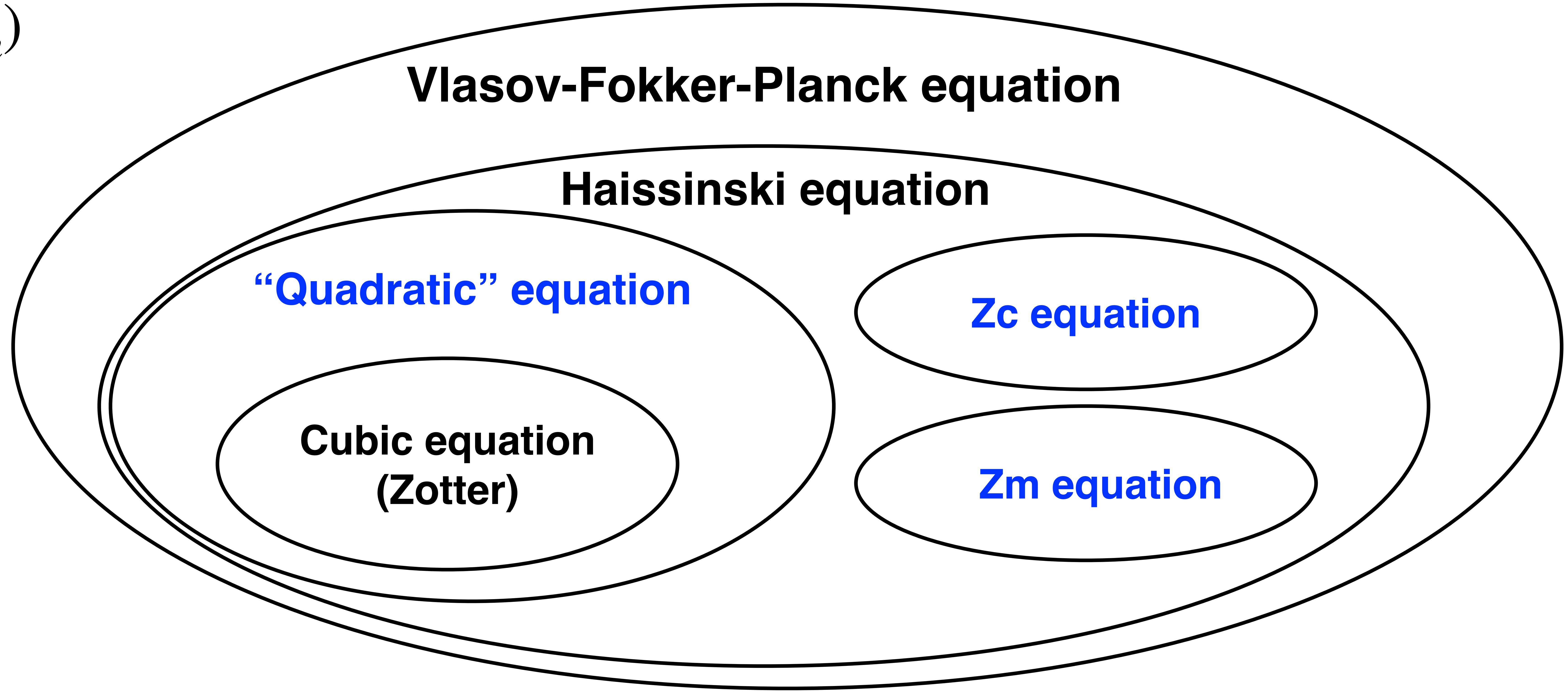
Haissinski equation: $$\lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2} - \frac{I}{\sigma_{z0}}\int_z^\infty dz'\mathbb{W}_\parallel(z')}$$

"Quadratic" equation: $$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}}Z_\parallel^{eff}(x) = 0$$

Zotter's equation: $$x^3 - x + \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta 0}^2(E/e)}\mathbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1} = 0$$

Zc equation: $$z_c(I) = I\sigma_{z0}\kappa_\parallel$$

Zm equation: $$z_m = I\sigma_{z0}\mathbb{W}_\parallel(z_m)$$



Subset diagram

Summary

- Zotter's equation is valid with assumptions
  - The longitudinal total impedance of the ring can be well approximated by a pure inductance.
  - The impact from the real part of the total impedance is negligible.
  - The potential well remains well quadratic so that the lengthened bunch is close to Gaussian.

  - Applicable cases: NSLS-II, Diamond (light sources with dominating inductive impedances from small discontinuities (bellows/flanges) and tapers (insertion devices)) [1]; SuperKEKB LER (colliders with dominating impedances from small-gap collimators).
  - Non-applicable cases: damping rings (no insertion devices or collimators, SLC damping ring with smoothed chamber is a good example), Future Circular Colliders (FCCs, resistive-wall impedance dominates because of large ring circumference).
- A simple equation for potential-well bunch lengthening is derived from Haissinski equation, useful for correlating impedance computations with simulations and beam-based measurements.

[1] V. Smaluk, NIMA 888, 22 (2018).

Backup

Equations derived from Hassinski equation

- Center of mass z_c
  - Starting from the differential equation instead

$$\frac{d\lambda_0(z)}{dz} + \left[\frac{z}{\sigma_{z0}^2} - \frac{1}{\eta\sigma_\delta^2}F_0(z)\right]\lambda_0(z) = 0 \longrightarrow \lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2} - \frac{I}{\sigma_{z0}}\int_z^\infty dz' \int_{-\infty}^\infty W_\parallel(z'-z'')\lambda_0(z'')dz''}$$

Trick: Integrate this equation over z

$$z_c = \int_{-\infty}^{\infty} z\lambda_0(z)dz \longrightarrow z_c(I) = I\sigma_{z0}\kappa_\parallel$$

Wake potential: $\mathbb{W}_\parallel(z) = \int_{-\infty}^{\infty} W_\parallel(z - z')\lambda_0(z')dz'$

Loss factor: $\kappa_\parallel = \int_{-\infty}^{\infty} dz\lambda_0(z)\mathbb{W}_\parallel(z) = \frac{c}{\pi}\int_0^\infty \mathbf{Re}[Z_\parallel(k)]h(k)dk$

  - z_c is sensitive to real part of impedance, simply proportional to loss factor $\kappa_\parallel$.
  - The most trivial way of measuring loss factor might be collecting data of power assumptions: RF power, temperature of cooling water, …

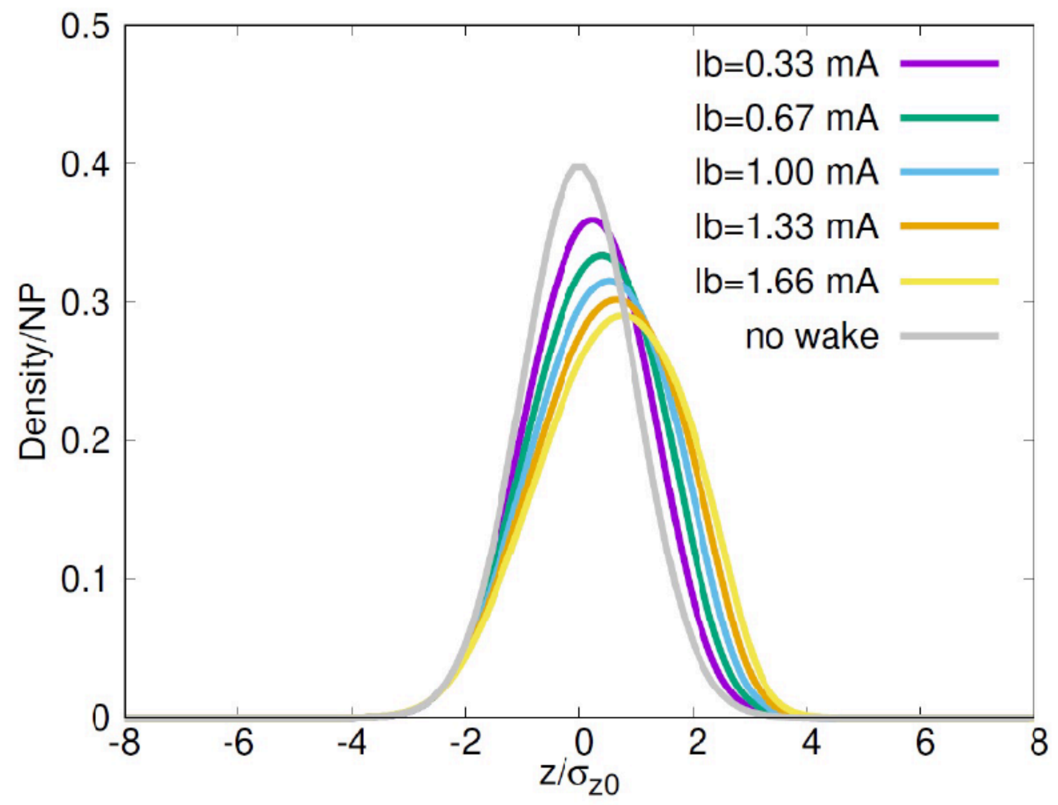
Equations derived from Hassinski equation

- Peak position of bunch profile z_m

$$\frac{d\lambda_0(z)}{dz} = 0 \longrightarrow z_m = I\sigma_{z0}\mathbb{W}_{\|}(z_m) \longleftarrow \frac{d\lambda_0(z)}{dz} + \left[\frac{z}{\sigma_{z0}^2} - \frac{1}{\eta\sigma_\delta^2}F_0(z)\right]\lambda_0(z) = 0$$

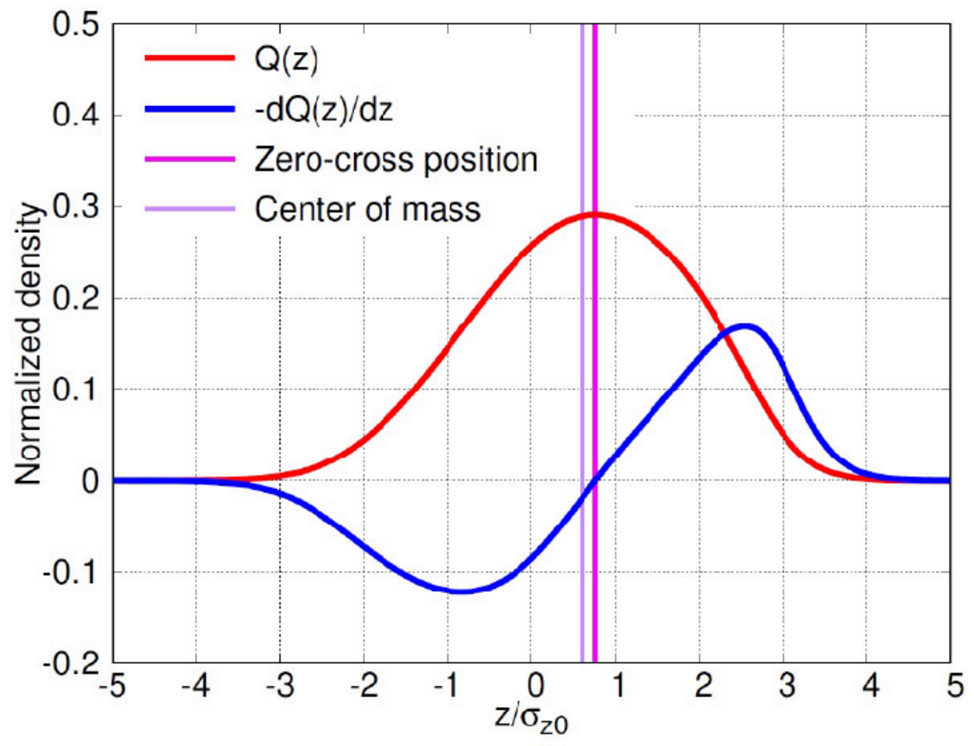
Trick: Take this term as zero

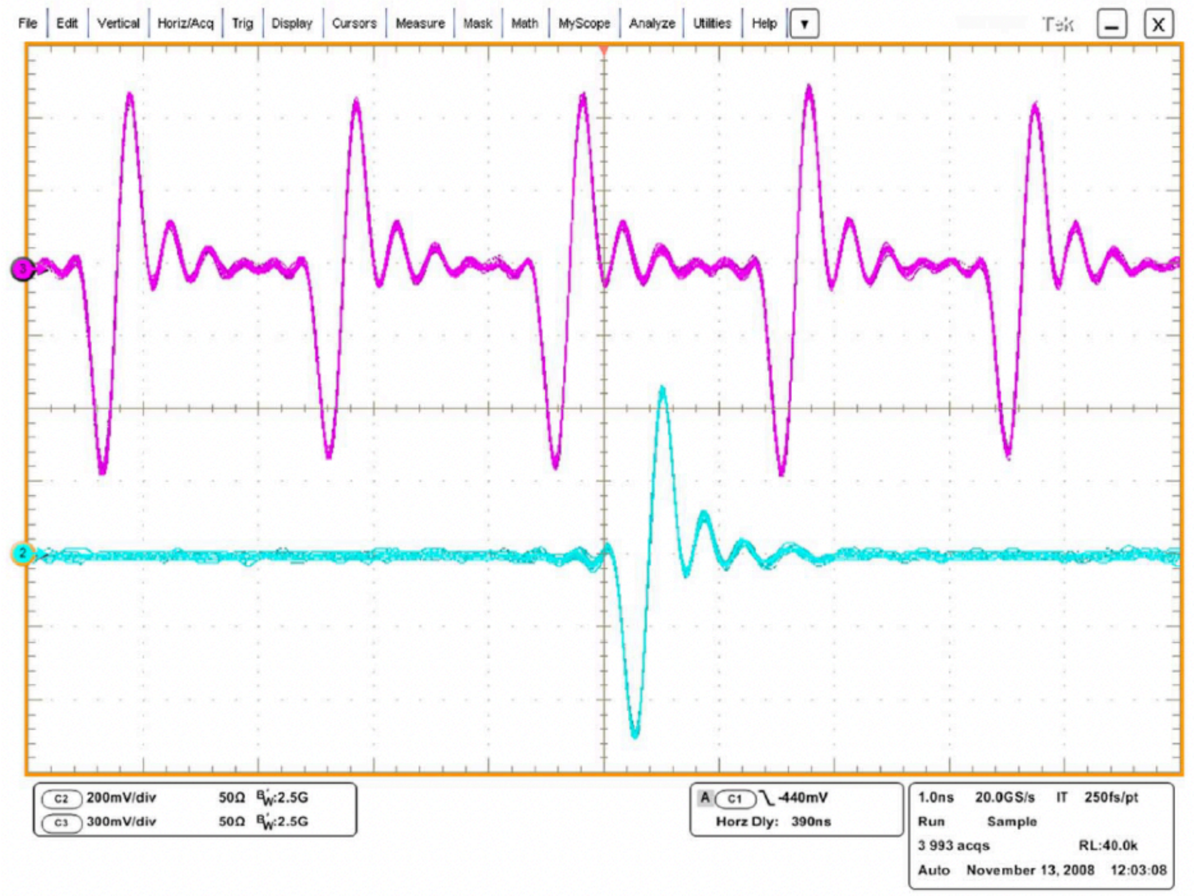
- z_m is sensitive to real part of impedance
- From measurement viewpoint, z_c and z_m have different meanings.
- Mostly, acc. physicists believe measuring z_c is trivial. They usually share simulation data of z_c to experimental experts.
- Practically, it is not trivial to measure z_c with good accuracy using synchrotron radiation light or using BPM signal.
- However, measuring z_m using BPM signal is possible (realized at SuperKEKB)



Simulated bunch profiles

BPM signal





Realistic BPM signal [1]

Simulated bunch profiles have tilt and shift in center-of-mass.
BPM signal: $i(t) = -dQ(t)/dt$
-> Zero-cross point of BPM signal is more relevant to peak of bunch profile

[1] T. Ieiri et al., NIMA 606 (2009) 248–256.

Equations derived from Hassinski equation

- Inverse problem of Haissinski equation
  - Wake potential extracted from simulated or measured bunch profile

$$\lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2} - \frac{I}{\sigma_{z0}} \int_z^\infty dz' \int_{-\infty}^\infty W_\parallel(z'-z'')\lambda_0(z'')dz''}$$

$$\downarrow$$

$$\mathbb{W}_\parallel(z) = \frac{\sigma_{z0}}{I}\left[\frac{d\ln\lambda_0(z)}{dz} + \frac{z}{\sigma_{z0}^2}\right]$$

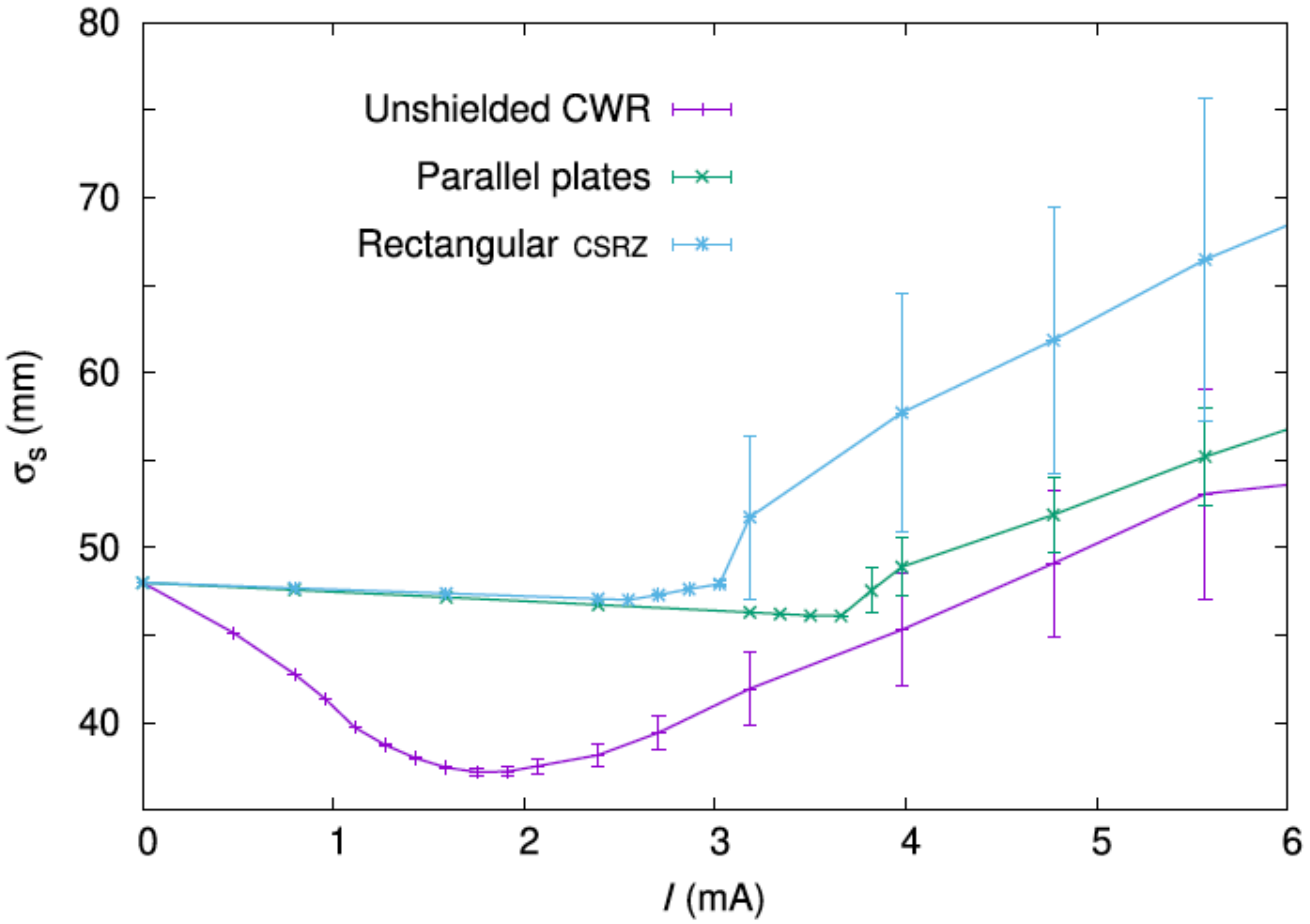
  - Impedance extracted from wake potential [1]

$$Z_\parallel(k) = \frac{\sigma_{z0}}{Ic^2\tilde{\lambda}_0(k)}\int_{-\infty}^{\infty}\left[\frac{d\ln\lambda_0(z)}{dz} + \frac{z}{\sigma_{z0}^2}\right]e^{-ikz}dz$$

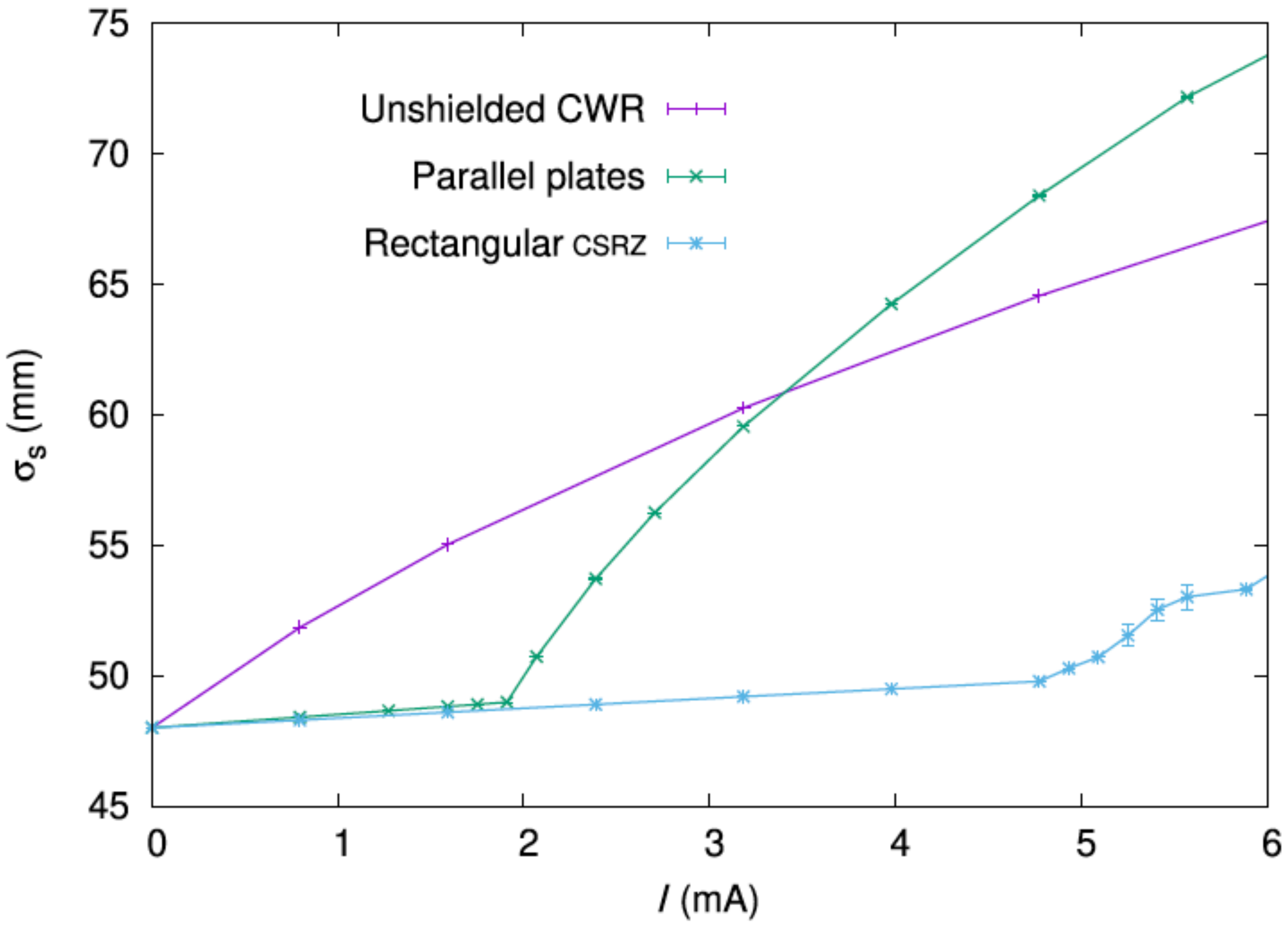
[1] A. Chao, Lectures on Accelerator Physics (World Scientific, 2020)

Some practical examples

- Bunch shortening by free-space CSR/CWR
  - Significant bunch shortening/lengthening for positive/negative momentum compaction in simulations for EIC ring electron cooler [1]
  - Practically, chamber shielding suppresses low-frequency CSR/CWR, such effects have not be observed in measurements



Positive α_c



Negative α_c

[1] A. Blednykh et al., PRAB 26, 051002 (2023).

Some practical examples

- Using BPM to measure beam phase
  - It’s not trivial to detect the center of mass using BPM signals
  - Detecting the peak position of the bunch profile using BPM signals (zero-cross point) was developed at SuperKEKB

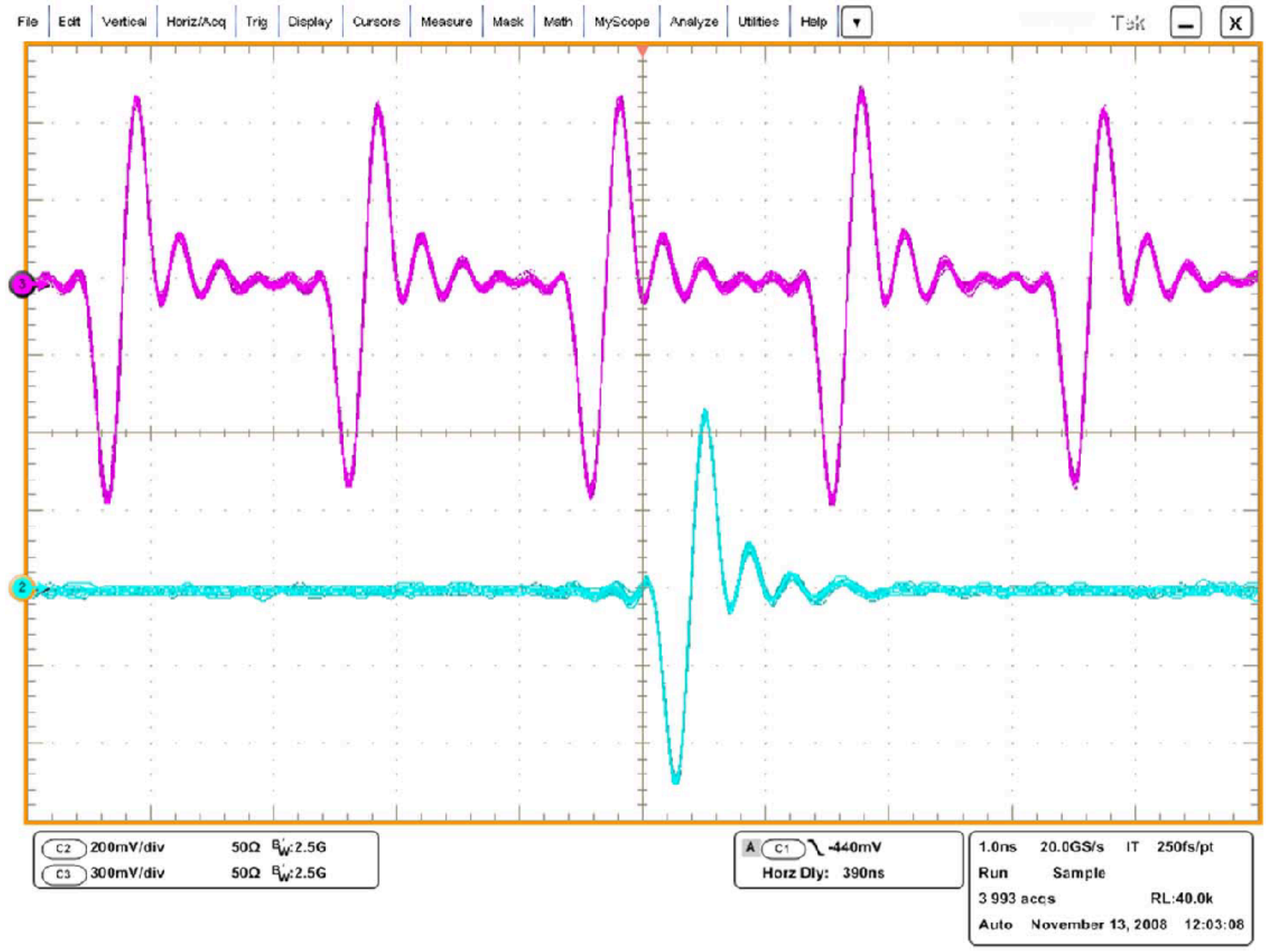
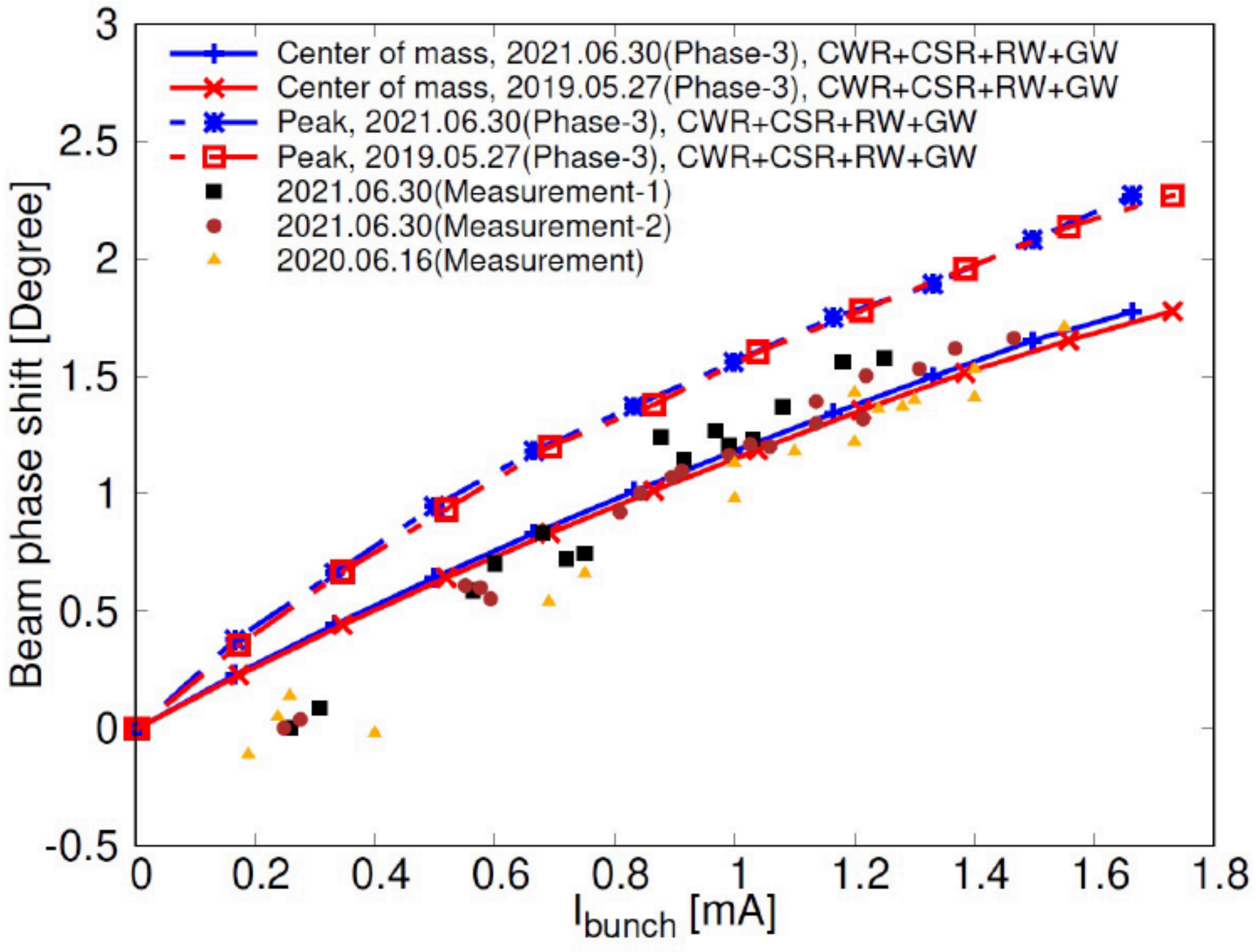
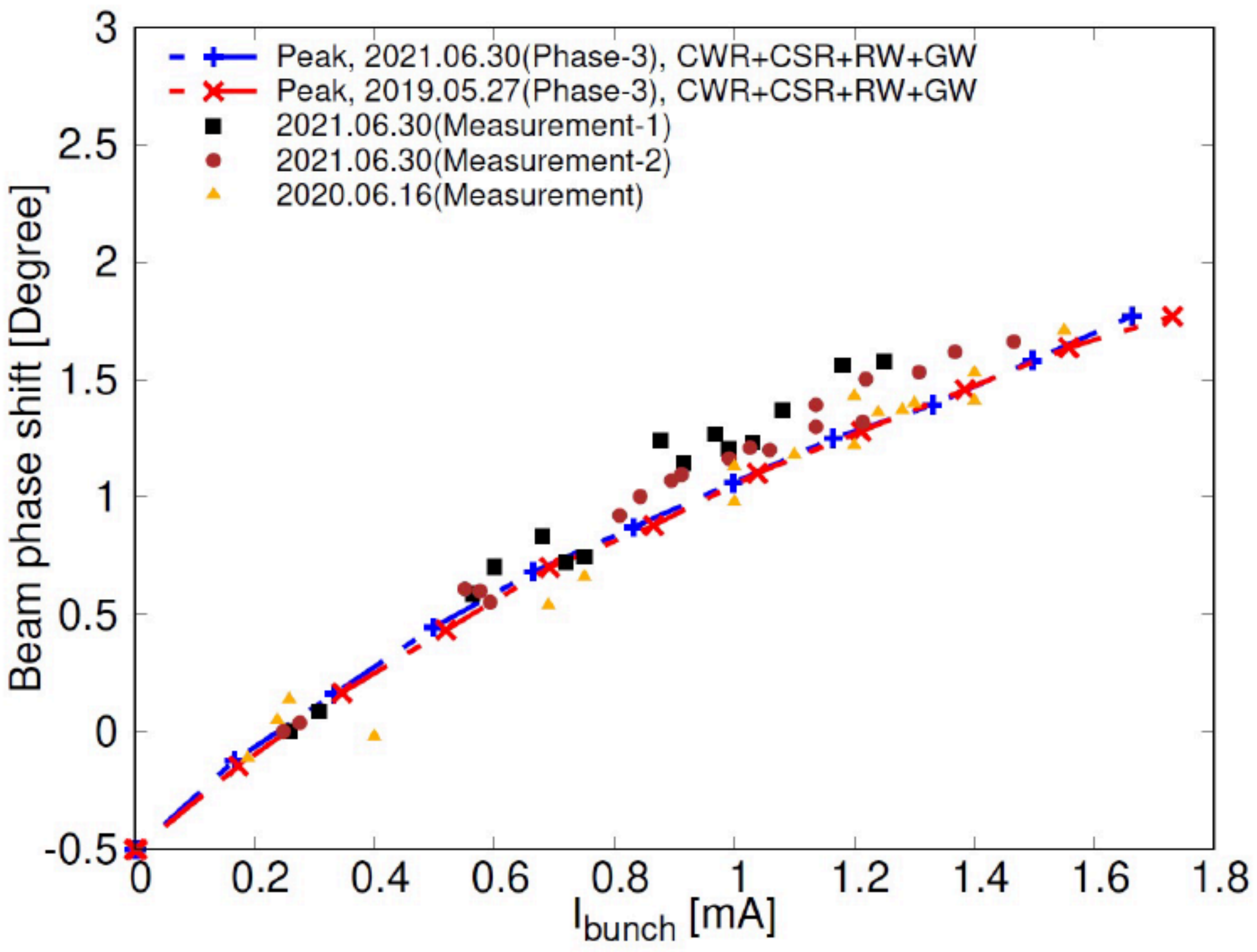


Fig. 10. Bunch signal of the electron beam picked-up by a button electrode. Upper trace corresponds to a part of a bunch train with a spacing of 2 ns, the bunch current is 0.5 mA. Lower trace corresponds to a gated bunch signal. The horizontal scale is 10 ns in full range or 1 ns/div. and the vertical scale is 300 mV/div. for the upper trace and 200 mV/div. for the lower one.

BPM signal measured at KEKB in 2008 [1]



Simulated center of mass did not fit the measured data well

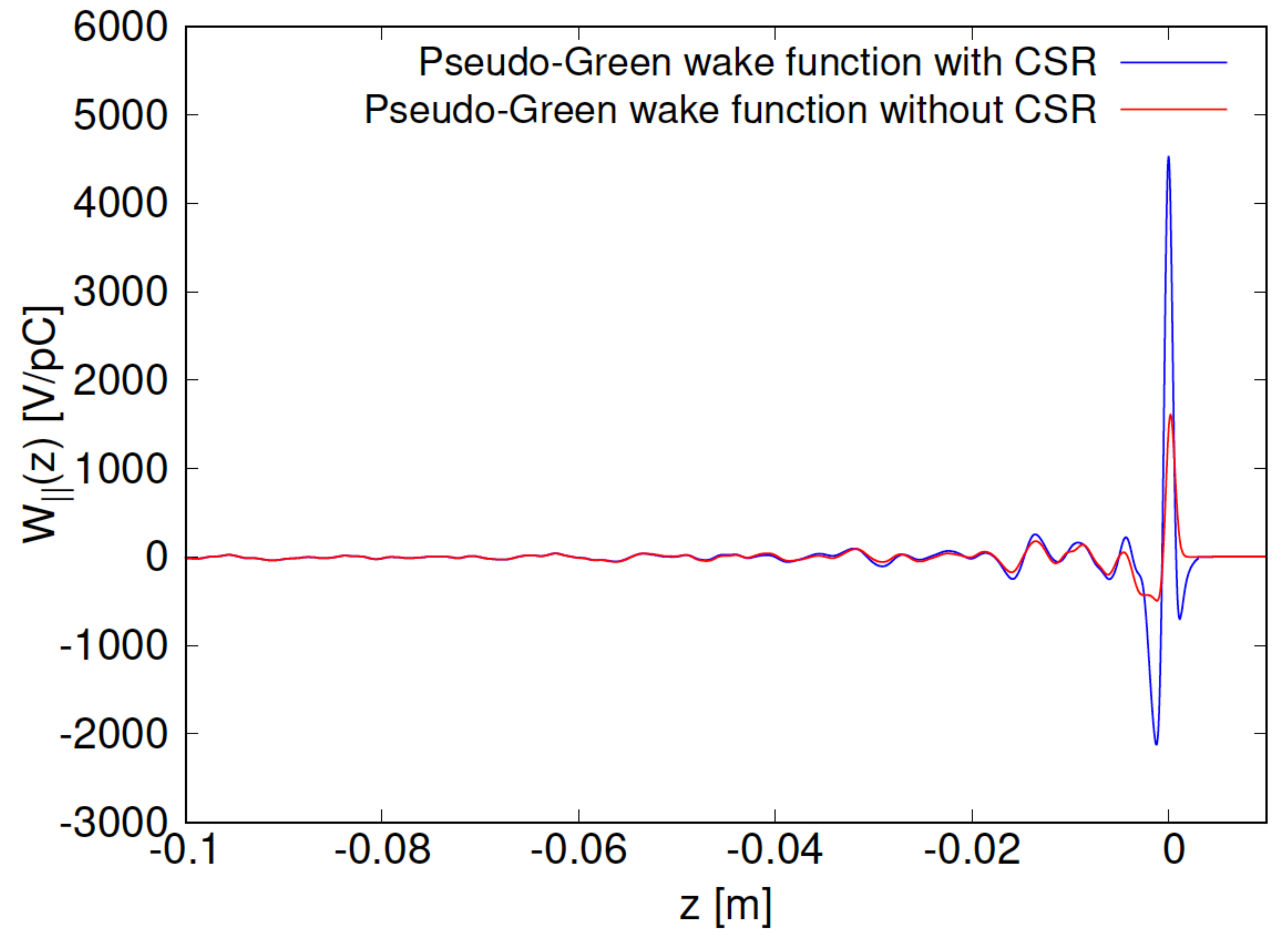


Simulated peak position did fit the measured data well, especially at low bunch currents

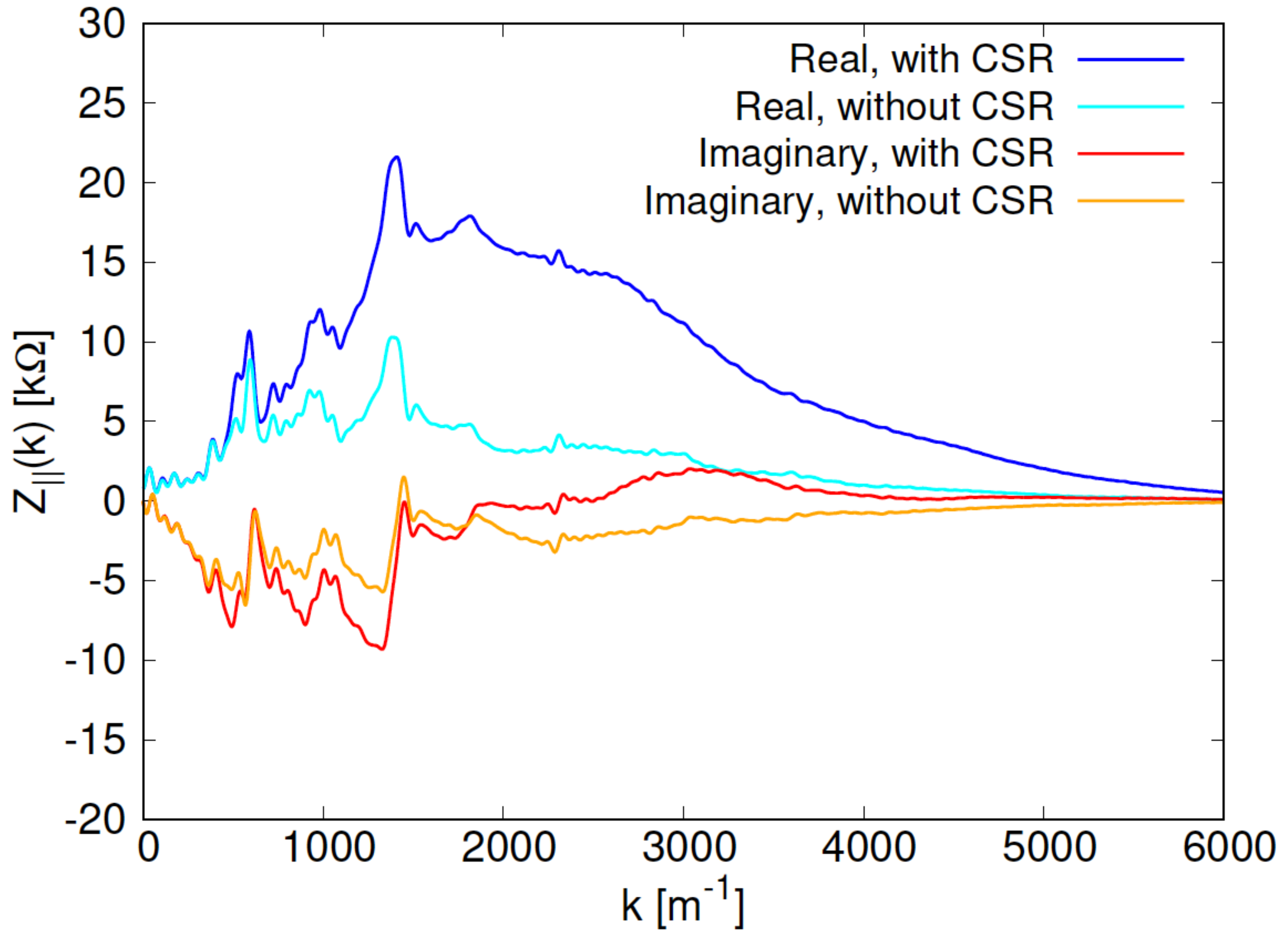
[1] T. Ieiri et al., NIMA 606 (2009) 248–256.

Some practical examples

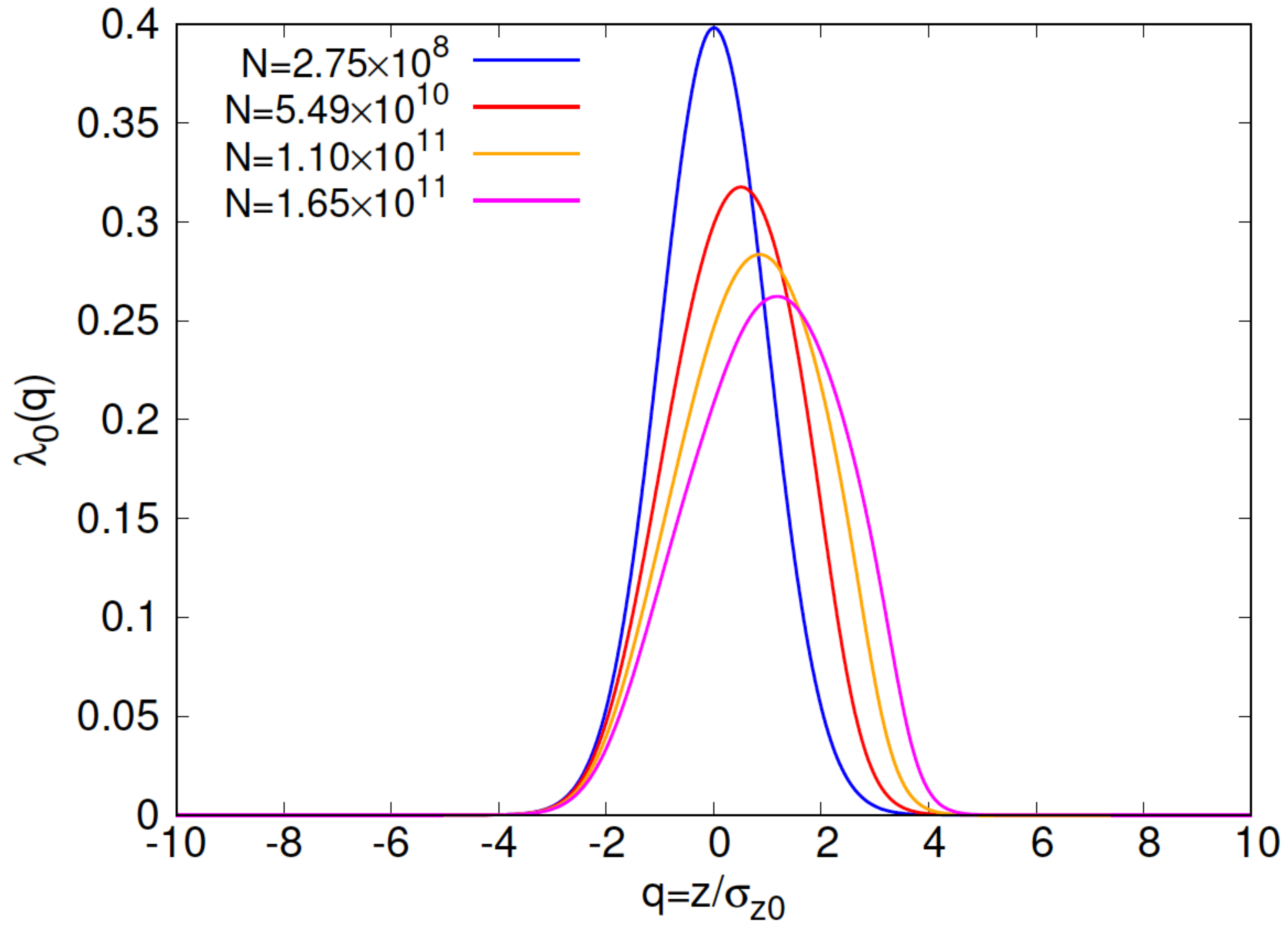
- SuperKEKB LER
  - Pseudo-Green function wakes constructed and used inputs of simulations



Pseudo-Green function wakes with 0.5 mm Gaussian bunch



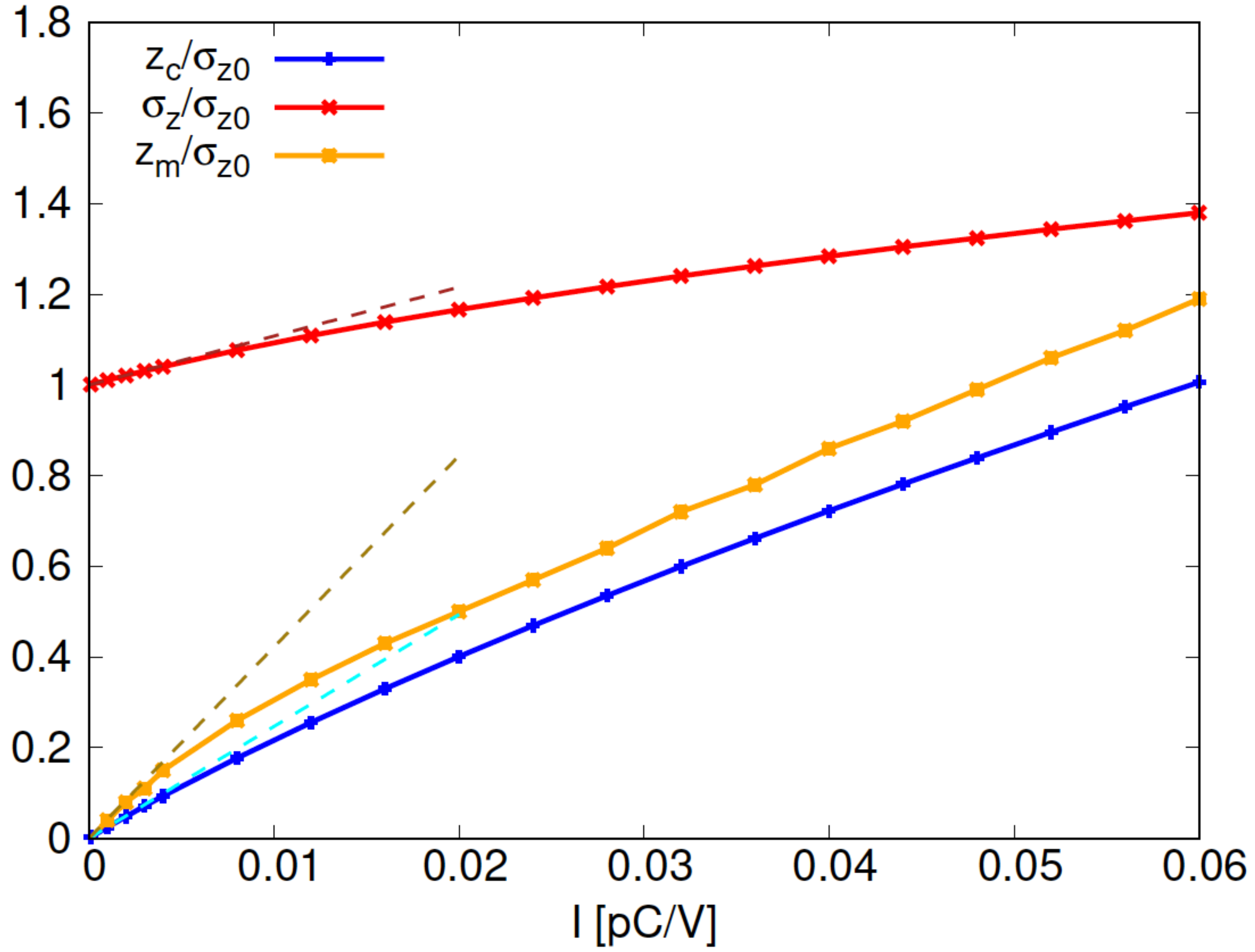
Fourier transform of short-bunch wakes



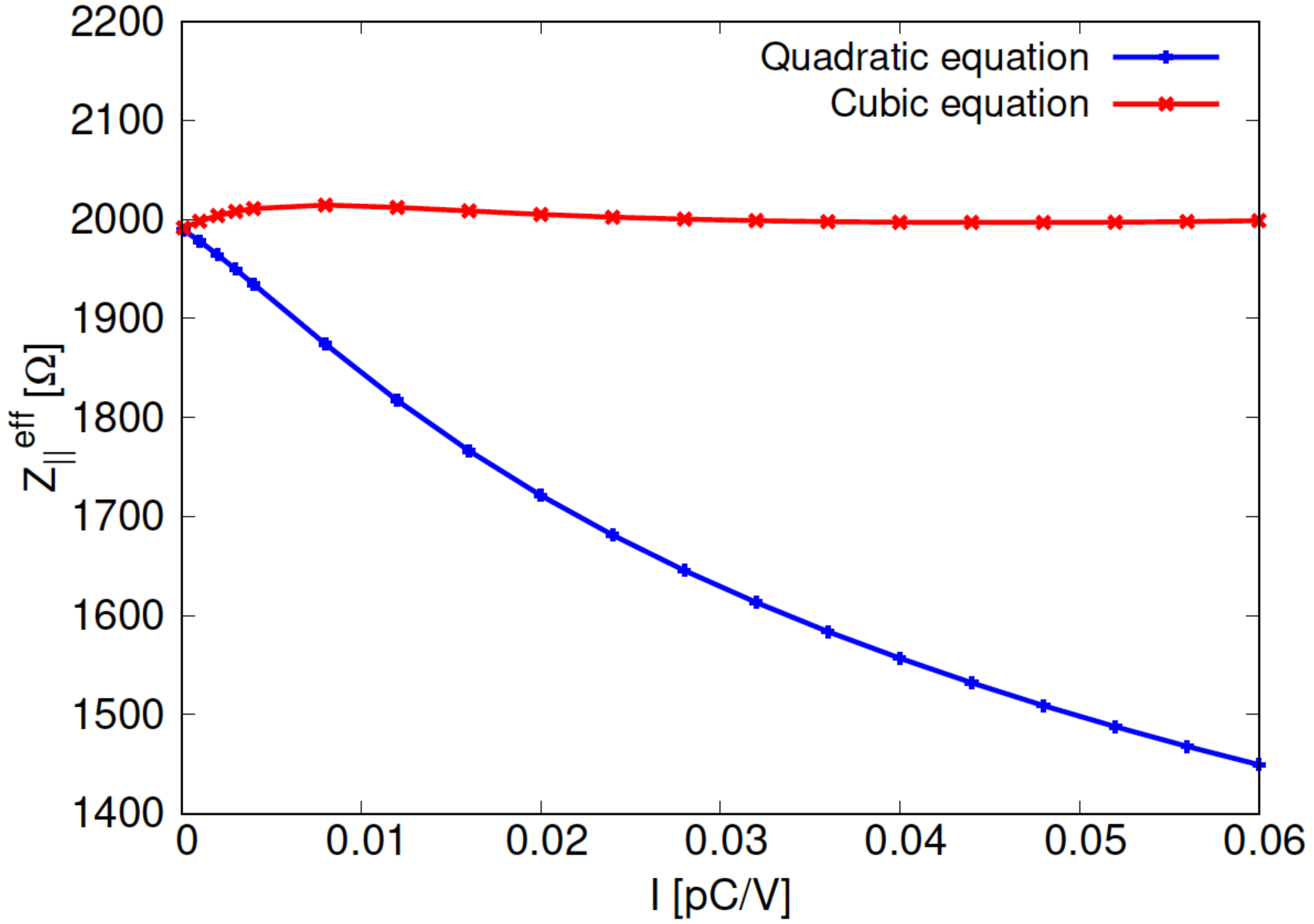
Haissinski solutions

Some practical examples

- SuperKEKB LER
  - Pseudo-Green function wakes constructed and used inputs of simulations



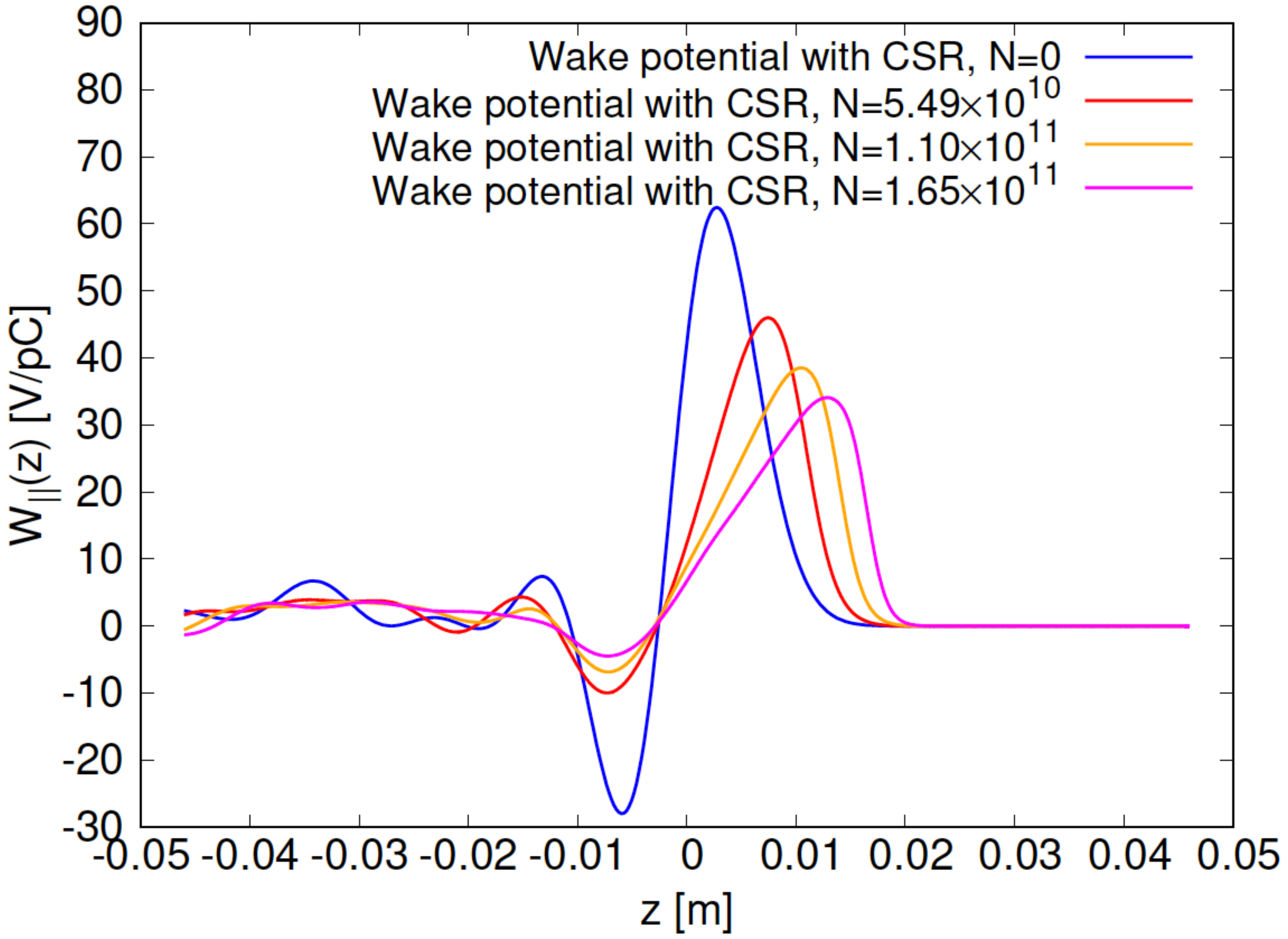
The slopes at zero current have clear meanings with given impedance and nominal bunch



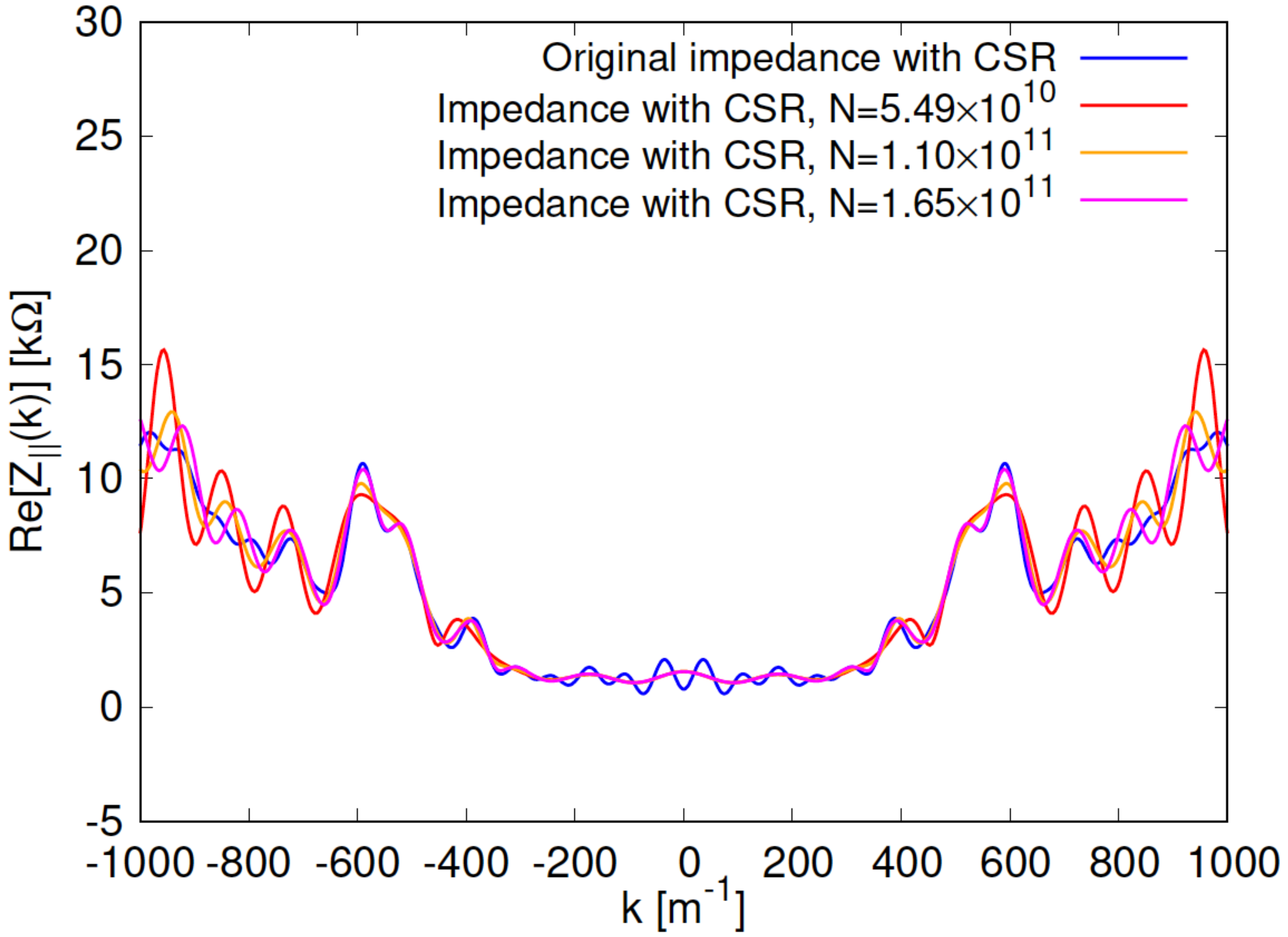
Effective impedance shows machine properties

Some practical examples

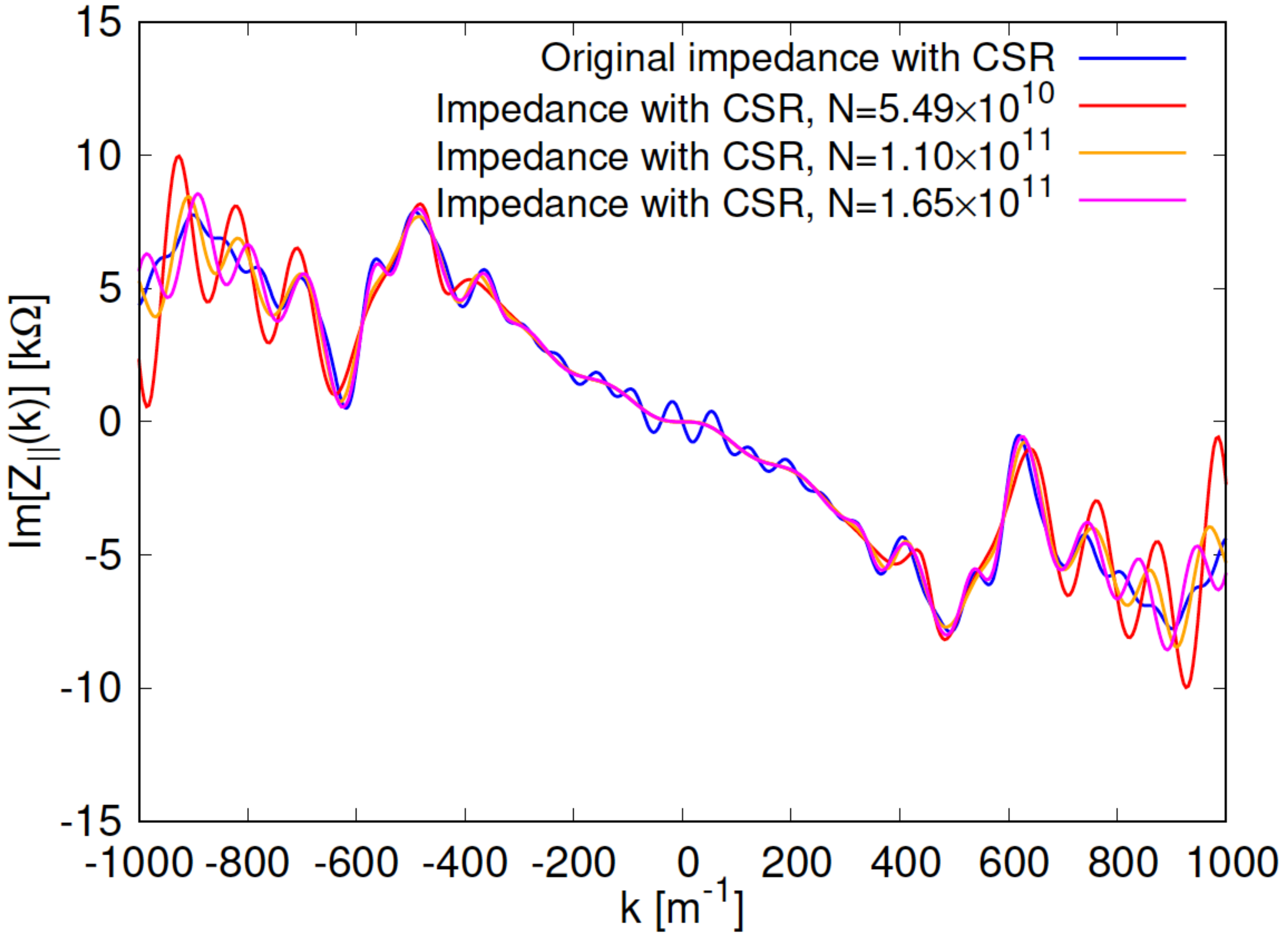
- SuperKEKB LER
  - Pseudo-Green function wakes constructed and used inputs of simulations



Wake potential with different bunch profiles



Real part of impedance extracted from Haissinski solutions



Imaginary part of impedance extracted from Haissinski solutions